WAVE RUN-UP ON STEEP SLOPES - MODEL TESTS UNDER RANDOM WAVES

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Report SR 2
August 1985
This report describes work carried out by Hydraulics Research under three contracts concerned with research on the stability and hydraulic performance of rubble mound structures:

(a) PECD 7/7/130, funded by the Department of the Environment (Water Directorate), nominated officer Mr R B Bussell;

(b) DGR 465/30, funded by the Department of Transport from April 1982 to March 1984 and thereafter by the Department of the Environment, nominated officer Mr A J M Harrison;

(c) Commission B, funded by the Ministry of Agriculture, Fisheries and Food, nominated officer Mr A Allison.

At the time of reporting this project, Hydraulics Research's nominated project officer was Dr S W Huntington.

Dr Ing L. Franco was supported during his period in the UK by the State University of Rome, and by a scholarship granted by the Italian National Research Council.

This report is published on behalf of the Department of the Environment and the Ministry of Agriculture, Fisheries and Food, but any opinions expressed are those of the authors only, and are not necessarily those of those ministries.

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ABSTRACT

In the design of coastal structures it is often necessary to calculate the levels to which waves will run-up on the front face of the structure.

This report summarises the results of a series of model tests conducted with random waves to measure the wave run-up and reflection performance of steep smooth and armoured slopes. These tests follow a literature review, published separately, on wave run-up on smooth and armoured slopes as used in the construction of breakwaters, seawalls and revetments. The results of this study are intended to assist those designing, constructing or maintaining such structures.

An understanding of the run-up performance of the seaward face of a seawall or breakwater, whether armoured and rough, or relatively smooth, is required to allow the designer to:

(a) estimate the crest level of the structure to permit the exceedance of a certain proportion of the waves for various alternative types of armouring;

(b) deduce suitable values for different wave conditions of a roughness factor used in estimating overtopping discharges, thus allowing the economic design of rear slopes.

The report presents measurements of run-up levels for smooth slopes, and for slopes armoured with tetrapods, antifer cubes, stabits, diodes and SHEDs. The effects of structure slope, wave steepness and spectral shape are considered, as are different measurement techniques. Various empirical expressions are fitted to the test results, and may be used for estimating typical run-up levels in preliminary design. Comparisons have been made with those prediction methods available, and a number of discrepancies between the results of the various methods have been explored. The report identifies an uncertainty in the prediction of run-up levels on smooth slopes of the order of 30%. The report presents measurements of random wave run-up on armoured slopes and compares results with those predicted from regular wave work. These simple comparisons appear to show relatively good agreement. Measurements of the wave reflection performance of the slopes tested are also presented.
Notation

A
B
C
a
b
c

$D_{50}$ median rock diameter
$g$ gravitational acceleration (m/s)
$H$ wave height, crest to trough
$H_o$ offshore wave height, in deep water
$H_s$ significant wave height of a steady sea state
$H_2$ wave height exceeded for only 2% of the waves in a steady sea state
$I_r$ Iribarren number, defined in 4.2
$I'_r$ modified Iribarren number, defined in 4.2
$K_r$ structure reflection coefficient, defined in 2.3
$L$ wave length
$L_o$ deep water wave length
$L_s$ wave length at the structure
$r$ nominal slope roughness coefficient
$R$ run-up, expressed as a height above static water level
$R_s$ significant wave run-up, mean of highest third run-up crests
$R_2$ run-up height exceeded by only 2% of run-up crests
$\bar{R}$ mean run-up height, arithmetic mean of all run-up crest heights
$S_r$ reflected spectral energy density
$S_i$ incident spectral energy density
$T_z$ mean zero crossing wave period
$T_p$ wave period of peak spectral energy
$\alpha$ structure slope angle to the horizontal
$\beta$ incident wave angle, wave crests to seawall
$\Delta x$ wave probe spacing
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1 INTRODUCTION

1.1 General

Wave energy incident upon a seawall or breakwater may be reflected, transmitted or dissipated. Waves reflected from such a structure will lead to increased wave activity in its vicinity, leading in turn to higher wave orbital velocities and hence bed scour, and to additional movements for vessels moored or navigating nearby. Waves transmitted over or through a breakwater designed for overtopping may not give rise to such problems, but wave energy transmitted over a seawall will cause flooding and/or damage to land behind or to the seawall itself. The object of good hydraulic design of such structures is to dissipate the majority of the incident wave energy in as economical a fashion as possible. The primary mechanism for the dissipation of wave energy is turbulence, often associated with wave breaking. Most storm waves can be induced to break on shallow smooth slopes, but will tend to reflect from smooth slopes built at the steeper angles dictated by the costs of construction of such structures. Wave energy may however also be dissipated by turbulent flow through the many voids formed around and between the armour, underlayer and core units of rubble mound seawalls or breakwaters. However, on both armoured rubble slopes, and smooth slopes, waves will also tend to run-up over the outer surface of the structure. If the structure crest is lower than the maximum run-up, the structure will suffer overtopping.

In the planning and design of coastal structures, especially seawalls, wave run-up and overtopping are often the primary hydraulic factors dictating the crest level of the wall. As the cross section area and the cost will increase approximately with the square of the structure height, accurate predictions of run-up performance are essential to the economic design of such structures. In the past designers often attempted to design the crest level of their structure high enough to prevent overtopping, by setting the crest level above a calculated maximum run-up level. This, however, presupposed that such a maximum run-up level could be determined. With a fuller understanding of the probabilistic nature of waves and water levels, it has become clear that overtopping cannot always be wholly prevented. The design of certain seawalls may therefore be based on the determination of an acceptable maximum overtopping discharge as discussed by Owen (1,2,3). For other structures, however, the design variable used to determine the crest level may be an extreme run-up level such as the 2% exceedance level used in the Dutch code(4).
This report considers prediction methods for the estimation of extreme run-up levels on coastal structures with relatively steep seaward faces, subject to random waves at perpendicular incidence.

1.2 Previous work

The prediction of run-up levels on smooth slopes is generally well documented, although most work has been based on tests using regular waves only. Some recent experimental work with irregular waves has been presented by Ahrens, who suggests a prediction method for run-up on smooth slopes under irregular waves (5). It is interesting however to note that this method has not been incorporated into the most recent edition of the Shore Production Manual (4th edition, 1984).

Prediction methods for armoured rough slopes suggested by the Dutch code (4), and by Ahrens (3), rely on applying a roughness reduction factor to run-up levels predicted for the equivalent smooth slope. Recent work by Losada & Gimenez-Curto (6), and by Allsop (7), has shown, however, that run-up levels on armoured rubble slopes are not well described by the application of a single roughness correction factor. Furthermore the general trends of run-up on armoured rubble slopes were not well described by the prediction methods commonly suggested. A careful review of the available design guidelines, and of recent research work was therefore initiated, and has been presented separately by Allsop, Franco & Hawkes (8). This review confirmed that run-up on armoured rubble slopes exhibits different trends from that on smooth slopes, and that the application of a simple reduction factor was liable to lead to some inaccuracy in the estimation of run-up levels. This was particularly so for the steeper structure slopes used for rubble mound construction. The review therefore recommended that model tests should be conducted to measure run-up on both armoured rubble slopes and smooth slopes under random wave attack. It was suggested that a number of different armour units should be tested, and that various probability distributions should be fitted to the random wave run-up levels measured. It was hoped that the results of these tests would then allow the derivation of empirical expressions for the prediction of run-up levels on both armoured rubble slopes and smooth slopes.

1.3 Outline of this study

Following from the literature review, a series of tests were devised to measure the run-up and reflection performance of rubble slopes armoured with units in two layers, tetrapods and antifer cubes, and in a single layer, stabits, diodes and SHEDS. Rock
armour was not chosen as the random nature of rock size and shape, together with the wide range of placing methods and densities, would have required a more extensive test programme than was possible within the time available. Smooth slopes were included to allow comparison of the results of the proposed test and measurement methods with the results of previously published prediction methods.

The particular concrete armour units selected were chosen to be typical of those used in rubble breakwater and seawall construction around the UK, and elsewhere. The tetrapod has been used widely around the world since its introduction in the early 1950s by the Neyrpic Hydraulic Laboratory, and is described by Danel, Chapus and Dhaillie. The tetrapod is normally laid in two layers to an essentially regular pattern. The antifer cube is a grooved and tapered cubic unit, and is also usually laid in two layers. It was first used at the French port of Antifer near Le Havre, and has since been used on a number of sites including the rehabilitation work at Sines, reported by Mol et al. Breakwater sections armoured with antifer cubes have been model tested by Allsop and Steele. Stabits have also been used widely on breakwater and seawalls, usually in a single layer laid in a pattern known as "brickwall". The development and use of the stabit have been described by Singh, and recent model tests of a stabit armoured breakwater have been reported by Owen, Steele and Allsop. The last two armour units considered in this study are both laid in a single layer, to a completely regular pattern. The development and use of the diode unit has been described by Barber and Lloyd. The SHED unit has been used on both breakwaters and seawalls around the UK, and in the Mediterranean and Arabian seas. Hollow cube units, including the SHED and the cob, have been considered by Wilkinson and Allsop, whilst details of model tests to determine the hydraulic performance of the SHED unit have been given by Allsop. Some test results from that study have been included in later chapters of this report.

The construction of the test sections, and the measurement techniques employed for wave reflections and run-up are discussed in chapter 2 of this report. The test results are presented separately, reflections in chapter 3 and run-up in chapter 4. Work fitting probability distributions to some of the measured data is described in chapter 5. Comparisons of the typical run-up levels measured on both smooth and armoured slopes are made with those predicted by a variety of methods in chapter 6. The use of these test results, and the conclusions of the study are given in chapter 7.
2 MODEL TESTS

2.1 Test facility

The model tests in this project were conducted in the deep random wave flume at Hydraulics Research. This flume, shown in Fig 2.1, is 52m long and is divided for much of its length into a central test channel, ending in a finger flume, and two side absorption channels. Splitter walls of graduated porosity are designed to minimise the level of re-reflected waves. The wave paddle is a buoyant wedge driven by a double-acting hydraulic ram. The random wave control signal is supplied by an HRS spectrum synthesizer described elsewhere by Fryer, Gilbert and Wilkie (16). An hierarchical system of PDP mini-computers is used to perform on-line analysis of all suitable analogue measurement signals using either statistical or spectral analysis programs. The principles of these measurement and analysis methods have been discussed by Dedow, Thompson and Fryer (17).

2.2 Test sections

The smooth and armoured slopes tested were constructed on a hinged test frame that had been used for the earlier study of the hydraulic performance of a single layer hollow cube armour unit, and is shown in Fig 2.2. This frame was hinged to a small toe wedge, allowing slopes of 1:1.33, 1:1.5 and 1:2.0 to be set. The rubble slopes were build on a perforated metal sheet, of 22% area porosity, 3mm diameter holes, supported on the frame. Rock underlayer was laid over the perforated sheet to a minimum thickness of 4 D₅₀ to support the armour. The thickness of the underlayer was adjusted so that the upper surface of each armour layer was at the level of the sides of the support frame. The underlayer was blended from crushed limestone to the following specification:-

<table>
<thead>
<tr>
<th>Sieve size (mm)</th>
<th>Fraction by weight (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-6</td>
<td>10</td>
</tr>
<tr>
<td>6-12</td>
<td>20</td>
</tr>
<tr>
<td>12-18</td>
<td>10</td>
</tr>
<tr>
<td>18-25</td>
<td>60</td>
</tr>
</tbody>
</table>

A size grading of the resulting blend is shown in Fig 2.3.

For smooth slopes the frame was covered by a wooden sheet, but for the armoured rubble sections, five different armour units were used. The construction of these armoured rubble test sections is summarised below, in model dimensions. Details of the hollow cube armoured slopes tested earlier are included.
The wooden board used for smooth slopes, and the sides of the frame for the rubble slopes were each marked off in painted intervals of 100mm width up the slope. These painted bars were used to judge run-up levels from the video recordings. In the earlier tests, the SHED units themselves, laid in a close regular pattern, were used to deduce run-up levels.

The tetrapods were laid in the conventional two layer fashion with lower layer units in herringbone pattern. The resulting armour pack containing 448 tetrapods is shown in Fig 2.4. Stabits were however laid in the single layer brickwall pattern used on a number of breakwaters and as tested by Owen, Steele and Allsopp (13). A total of 293 Mk III stabits were used on this test section, shown in Fig 2.5. Antifer cubes are normally laid in two layers. Around 640 cubes were used on each test slope, but at the steeper slopes of 1:1.33 and 1:1.50 it was not possible to maintain the original packing density, and a slightly denser placing resulted. This section is shown in Fig 2.6. The diode units were laid in a single layer to the regular pattern shown in Fig 2.7. The SHED units had also been laid closely packed in a single layer, Fig 2.8.

### 2.3 Measurement of wave reflections

The measurement and analysis of wave reflections is best understood in terms of sine waves. A certain proportion of the energy of a sine wave incident on a slope will be reflected as a sine wave of the same...
2.4 Measurement of wave run-up

period, but of a lower height. The coefficient of reflection, $K_r$, may be defined as the reflected wave height divided by the incident wave height. If irregular waves are regarded as the sum of sine waves of different frequencies, then the reflection coefficients can be calculated for each frequency considered in the incident wave spectrum. The reflection coefficient, $K_r$, may then be defined for any frequency band width in terms of the reflected and incident energy densities in that band width, $S_r$ and $S_i$ respectively:

$$K_r = \frac{S_r}{S_i}^{\frac{1}{2}}.$$  

In this study wave measurements were made using two wave probes, separated by a distance $\lambda$. The incident and reflected wave spectra cannot be measured directly, but are calculated in an analysis program devised by Gilbert and Thompson(18), based on the method of Kajima(19).

The analysis method calculates values of $K_r$ over a wide range of frequencies, but the method is only valid over a restricted band related to the probe spacing. Two different probe spacings were therefore used for each test, allowing a wide range of wave frequencies to be covered.

Tests to measure wave reflections were run with the wave synthesizer set to produce short sequences. The wave measurement and analysis computer was then linked to the synthesizer, to allow analysis of precisely one sequence of waves. This allows the spectrum to be described without statistical uncertainty.

Run-up measurements were made by two different methods. The first of these used manual analysis of video recordings of waves on the slope. Use of a video camera mounted above and approximately normal to the slope had been tested successfully before. The analysis of the video recordings was performed by counting the total number, and hence proportion, of run-up crests exceeding certain fixed levels on the slope.

The second method used run-up strips or probes attached to the test section support frame, on the smooth slope, or immediately above the upper surface of the units on the armoured slopes. These run-up gauges operated on the same principle as the conventional twin-wire wave probes. They were carefully calibrated at each slope angle and were found to give a linear response to a very high standard.
The signal from the run-up gauge was logged by the mini computer system, and analysed by two methods. A statistical analysis program ranked departures from a fixed (given) level, in this instance static water level. This program produced histograms of the number of run-up crests falling into certain bands. The analogue output from the run-up gauge was also run to a bank of overtopping detector modules set to register exceedance numbers for given levels. In most of the tests too few of these counters registered run-up crests to allow the results to be used, except as a check on the correct function of the video and run-up gauge methods.

3 WAVE REFLECTIONS, TEST RESULTS

3.1 Analysis and presentation of results

The method used for the measurement and analysis of wave reflections have been covered briefly in section 2.3 above, and are discussed in greater depth by Gilbert and Thompson(18). The presentation of the results follows directly from the analysis method in which the coefficient of reflection is calculated at each of a number of frequency bands. The results may then be plotted as values of reflection coefficient, Kr against wave frequency, f. It should be noted that these frequency values are in model terms and have not been scaled.

The analysis technique does however assume that energy is not shifted from one frequency band to any other. However, in wave breaking, an incident long period wave may well give rise to a number of smaller and much shorter waves. If these short waves reflect, the analysis may calculate a greater coefficient of reflection for the high frequency short waves than is due to the incident waves of that frequency. A lower value of Kr may similarly be calculated for the low frequency, long period, waves. In some circumstances, values of Kr greater than unity may be calculated.

In considering the test results, account must be taken of the influence of such non-linearities, and the underlying trend must be identified. The results of the reflection measurements in this study are presented in Fig 3.1-5, and to those results have been applied a sketched trend line for each slope tested. These lines are not intended to represent the mean lines through the measured points, but to illustrate the likely limiting trends. In particular, where waves break at or on the test slope, the longer low
frequency waves may reflect partially as shorter high frequency waves. In these circumstances some measurements may suggest low values of $Kr$ at the lower frequencies, and high values at the higher frequencies. This shift of energy from low frequencies has been ignored in estimating the trend line, as it will only occur when the long waves are of sufficient steepness to break, and not when long waves of relatively low steepness are present. It is these longer waves that are of particular relevance to harbour design, and it is believed that the higher values of $Kr$ indicated by the trend lines shown are more realistic than would be given by a simple mean line.

3.2 Smooth slopes

The results of the measurement of the reflections from smooth slopes are summarised in Fig 3.1. The graphs for each slope are similar to those measured previously and shown by Wilkinson & Allsop(15) and Allsop(7). At cot $\alpha = 1.33$, $Kr$ remains around 1.0-0.9, with a very slight departure from that presented earlier(15). At the two shallower slopes, trends similar to those seen before were apparent. At cot $\alpha = 1.50$, values of $Kr$ varied from around 0.9 at 0.6 Hz down to about 0.8 at 1.0 Hz. In the main, test conditions on these two steeper slopes produced waves that surged up and down the slope without significant wave breaking. At cot $\alpha = 2.0$, the character of wave activity on the slope started to change, and some waves plunged and broke on the slope. Some indication of this is shown by the reflection characteristics of this slope, with $Kr$ varying from around 0.8 at 0.6 Hz down to about 0.55 at 1.0 Hz.

3.3 Armoured slopes

The armoured slopes all exhibit less reflections than do the equivalent smooth slopes. Results for these slopes are summarised in Figs 3.2-5, covering tetrapod, antifer cube, stabit and diode armoured slopes respectively.

Reflections from tetrapod armoured slopes are shown in Fig 3.2. As on smooth slopes, the steeper structure slopes give rise to higher reflection coefficients. For cot $\alpha = 1.33$, the reflection coefficient $Kr$ over the frequency range 0.6-1.0 Hz was around 0.35, this reduced to 0.30 for cot $\alpha = 1.5$, and at cot $\alpha = 2.0$ it reduced further to around 0.25.

The results for antifer cube armoured slopes shown in Fig 3.3 are far less clear. The scatter of results for cot $\alpha = 1.33$ and 1.50 was extreme, making it impossible to identify any clear trend line. This unexpected degree of scatter may be due to the many reflecting faces of the cubes as laid on the slope.
At cot $\alpha = 2.0$ a more coherent trend emerges, with reflection characteristics that closely match those measured on the equivalent tetrapod slope.

Measurements with the stabit armoured slopes produced a coherent set of reflection characteristics, shown in Fig 3.4. For all three slope angles reflections showed the same trends as were measured for the tetrapod slopes.

The diode armoured slopes were tested at the steeper two angles only. The reflection characteristics shown in Fig 3.5 exhibit slightly lower reflection levels than were measured for the tetrapod or stabit armoured slopes.

**4 WAVE RUN-UP, TEST RESULTS**

**4.1 Analysis methods**

Results of the two primary methods of run-up measurement discussed in 2.4 above, video recordings and run-up gauges, have been analysed in essentially the same ways. In both instances the results of the measurement of run-up on a particular slope, under a particular incident sea state, were tabulated as values of run-up levels with the corresponding numbers and/or proportion of run-up crests that exceeded each of those levels. At a very early stage of analysis in this study graphs were plotted of exceedance probability against run-up level, on linear axis. Such a method of presentation compressed the principal area of interest, extreme run-up levels, and did not lend itself to the easy comparison of theoretical probability distributions with the test results. A number of alternative approaches were therefore devised. Methods of fitting various probability distributions to the data are discussed later in Chapter 5 of this report, but are not covered in detail here. In this chapter the emphasis is primarily on the calculation of representative run-up levels from each of the measured data sets. The levels chosen, $R_2$ and $R_8$, were selected by reference to previous work, in particular the Dutch code$^{(4)}$ and work by Ahrens$^{(5)}$ and Allsop$^{(7)}$. For the analysis of relative run-up levels, two techniques have been used:

(a) Direct interpolation

Values of $R_2$ and $R_8$ were interpolated directly from the table of run-up levels at the 2% and 13.53% exceedance values respectively. This was the most direct method of determining typical run-up levels, and gave the most satisfactorily defined values of $R_2$ and $R_8$. The calculated
values were however directly affected by the scatter of the measured results, which was substantial in some data sets. In those instances other methods of deriving values of $R_2$ and $R_s$ were used. In general, however, direct interpolation was used whenever possible in this study.

(b) Fitted probability distribution

In a previous study\(^7\), probability distributions of the general form of a Rayleigh distribution were fitted to the measured run-up levels, for each of the sea states tested. This procedure may be illustrated by plotting values of $R_1^2$ against values of $-\ln Q(R_1)$, where $Q(R_1)$ is the exceedance probability of any level, $R_1$. A straight line was then fitted to the transformed data set by a simple regression analysis, giving values for coefficients $A$ and $B$ in an equation of the form:

$$R_1^2 = B - A \ln Q(R_1) \quad (4.1)$$

Values of $R_2$ and $R_s$ were then calculated from this equation for values of $Q(R_1)$ of 0.020 and 0.1353 respectively. The use of this method clearly pre-supposes that the Rayleigh distribution is indeed a good fit to the measured data, and that calculated values of $R_2$ and $R_s$ therefore reproduce accurately the trends of the run-up distribution.

It will be clear from the above that these analysis methods may each lead to different estimation of run-up levels for the same data set. Furthermore the two principal methods of measuring run-up, video recordings and run-up gauges, will also each lead to different sets of results. In the further consideration of the run-up measurements, the results derived by direct interpolation from the run-up gauge output will be regarded as the primary data set, to which the results of other methods will be compared where appropriate.

4.2 Presentation of run-up results

The principal factors affecting the run-up level of a wave on a simple slope may be listed:-

- Wave height $H$ or $H_s$
- Wave length $L$, or period $T_z$ or $T_p$
- Structure slope, $\cot \alpha$
- Incident wave angle $\beta$ ($\beta = 0^\circ$ in this study)
- Slope roughness $r$ ($r = 1.0$ for smooth slopes).
It has been noted in previous work that wave run-up levels of themselves are not directly suitable for further analysis, but are better expressed as dimensionless or relative run-up levels, such as $R_2/H_s$ and $R_s/H_s$. Similarly the principal input parameters, wave height and period, and the structure slope angle, have been successfully characterised by a number of authors by the dimensionless surf similarity parameter, or Iribarren number, $I_r$. This has been defined for regular waves as:

$$I_r = \tan \alpha/(H/Lo)^{\frac{1}{2}} \quad (4.2)$$

For random waves a similar parameter may be defined, the modified Iribarren number, $I_r'$:

$$I_r' = \tan \alpha/(H_s/L_p)^{\frac{1}{3}} \quad (4.3)$$

where $L_p = g T_p^2/2 \pi$.

The results of the test measurements have therefore been expressed as relative run-up values, and have been plotted against the modified Iribarren number.

### 4.3 Smooth slopes

Most run-up measurements in this study were made using run-up gauges. Relative run-up levels on the smooth slopes, derived by direct interpolation from the analysis of run-up gauge output, are shown in Fig 4.1. As described above, relative run-up levels are plotted against the irregular Iribarren number $I_r'$. Four sets of data are shown in this figure. Values of $R_2/H_s$ and $R_s/H_s$ are shown for each of the spectral types used in the tests. For significant run-up, $R_s/H_s$, the different spectral types appear to have no discernable effect on the levels measured. Simple regression analysis has allowed the fitting of straight lines. For the JONSWAP spectra:

$$R_s/H_s = 2.13 - 0.09 I_r' \quad (4.4)$$

and for the Moskowitz spectra:

$$R_s/H_s = 2.11 - 0.09 I_r' \quad (4.5)$$

These may clearly be described by a simple straight line over the range considered ($2.8 < I_r' < 6.1$) for both spectra:

$$R_s/H_s = 2.11 - 0.09 I_r' \quad (4.6)$$

The data for the 2% exceedance run-up level does not show quite such good agreement, but even if there were no spectral shape effect, this would not be particularly surprising. The value of $R_2$ is not as
accurately defined by any particular data set as the value of $R_S$, and greater scatter in the calculated values of $R_2$ might therefore be expected. For the JONSWAP spectra:

$$R_2/\text{Hs} = 3.35 - 0.18 \text{Ir}$$

and for the Moskowitz spectra:

$$R_2/\text{Hs} = 3.53 - 0.25 \text{Ir}$$

A careful examination of these two data sets suggests that this difference is not of great significance, and that $R_2/\text{Hs}$ may be described by a single expression fitted to both data sets:

$$R_2/\text{Hs} = 3.39 - 0.21 \text{Ir}$$

Equations 4.6 and 4.9 are shown as the trend lines in Fig 4.1. It should be noted that the data is scattered, and that insufficient data is available to determine statistical confidence limits. The equations calculated are also only valid over the range of conditions considered ($2.8 < \text{Ir} < 6.1$).

The video recording technique discussed earlier was also used to measure run-up levels. Levels calculated by direct interpolation from these measurements are shown in Fig 4.2, and are compared with results derived using this technique in an earlier study. The two sets of results compare well, and demonstrate similar trends for $R_2$ and $R_S$ as those seen in Fig 4.1. Again a simple regression analysis has been used to fit straight lines to the data sets in Fig 4.2, giving:

$$R_S/\text{Hs} = 2.50 - 0.16 \text{Ir}$$

$$R_2/\text{Hs} = 4.16 - 0.31 \text{Ir}$$

To test further the comparability of the results of this study with the earlier work, run-up levels were also calculated using Rayleigh probability distributions fitted to the run-up probe output, as outlined in 4.1b above. The results of this exercise are shown in Fig 4.3. Again the results for different spectral types are plotted separately. A careful examination of results from the two spectral types again showed no significant differences, and simple linear regression gave virtually identical results. The lines fitted to the combined data sets may be summarised:

$$R_S/\text{Hs} = 2.34 - 0.12 \text{Ir}$$

$$R_2/\text{Hs} = 3.56 - 0.23 \text{Ir}$$
The significant run-up levels $R_s$, were very close to those deduced directly from both the run-up gauge output and the video recordings (Figs 4.1 and 4.2 respectively). At the more extreme 2% exceedance level the agreement between results from the different methods is not so good. Those derived from the run-up gauge output, Figures 4.1 and 4.3, show close agreement, indicating that the Rayleigh probability distribution gives a reasonable description of the results. The results for 2% exceedance level $R_2$, derived from the video recordings are generally higher than those derived from the run-up gauge output. This discrepancy might indicate a tendency for the observer to over-estimate the numbers of extreme run-up crests, or possibly for the run-up gauges to under-estimate their occurrence.

Implicit in the method of presentation used in Figs 4.1-3 is the assumption that the effect of armour slope angle, $\alpha$, is indeed fully described by the tan $\alpha$ term in the irribarren number. The validity of this assumption has been explored by re-presenting the run-up levels derived from run-up gauge output by direct interpolation, as shown in Fig 4.1, plotting data for each slope angle separately in Fig 4.4. The regression lines shown are those calculated for the complete data sets and given earlier as equations 4.6 and 4.9. Consideration of the data in Fig 4.4 suggests that the data sets for each slope angle are indeed well described by the parameter chosen, $R/Hs$ and Ir', within the normal scatter of such measurements.

4.4 Armoured slopes

On the armoured rubble slopes tested in this study, the wave run-up levels were measured at, or as close as possible to, the upper surface of the armour layer. Both run-up gauges and video recording measurement techniques were used, as were both analysis methods outlined in 4.1 above. The two most satisfactory sets of results were those derived by direct interpolation from the run-up gauge output for the slopes armoured with tetrapods or with antifer cubes. The results for these two armour units are shown in Figs 4.5 and 4.6. To each set of results have been fitted curves of the general form proposed by Losada and Gimenez-Curto by an iterative regression method. For the tetrapod armoured slopes these regression lines may be given by:

$$R_s/Hs = 1.32 \left[1 - \exp (-0.31 \text{ Ir}')\right]$$  \hspace{1cm} (4.14)

$$R_2/Hs = 1.83 \left[1 - \exp (-0.30 \text{ Ir}')\right]$$  \hspace{1cm} (4.15)
For the antifer cube armoured slopes similar lines may be given by:-

\[ \frac{R_s}{H_s} = 1.07 \left[ 1 - \exp\left(-0.45 \frac{I_r'}{I_r}\right) \right] \quad (4.16) \]
\[ \frac{R_2}{H_s} = 1.52 \left[ 1 - \exp\left(-0.34 \frac{I_r'}{I_r}\right) \right] \quad (4.17) \]

These lines each give a possible mean trend of the sets of data points. However, it has not been possible to ascribe statistical confidence limits.

Another general expression that may be used to characterise run-up on porous, rough slopes is that ascribed to CERC by Günbak\(^{(20)}\), which may be written:-

\[ \frac{R}{H_s} = \frac{A \frac{I_r'}{I_r}}{\frac{I_r'}{I_r} + B} \quad (4.18) \]

Günbak gives run-up levels measured on rock slopes under regular waves, and fits an expression that may be written:-

\[ \frac{R}{H} = \frac{1.6 \frac{I_r'}{I_r}}{\frac{I_r'}{I_r} + 2.0} \quad (4.19) \]

Expressions of this general form have also been fitted to the tetrapod slope results considered above:-

\[ \frac{R_s}{H_s} = \frac{2.346 \frac{I_r'}{I_r}}{\frac{I_r'}{I_r} + 6.50} \quad (4.20) \]
\[ \frac{R_2}{H_s} = \frac{3.575 \frac{I_r'}{I_r}}{\frac{I_r'}{I_r} + 7.887} \quad (4.21) \]

Run-up behaviour on the armoured slopes was also measured using the video recording technique. For some slopes, measurements using the run-up gauges were not available. It was therefore felt to be important to compare run-up measurements using both gauges and video recordings when possible to allow the use of results from the video recordings only if necessary.

Both analysis methods were used for the tetrapod armoured slopes. Run-up levels derived by direct interpolation from the gauges have already been discussed, those derived by the same method from the video recordings are shown in Fig 4.7, and those calculated by fitting the Rayleigh distribution to the video results are shown in Fig 4.8. Both sets may be compared with those in Fig 4.5. It is clear, however, from this comparison that agreement between the two measurement methods is not good, particularly in comparison with the good agreement shown for smooth slopes. The scatter of data values derived from the video recordings for the tetrapod slopes was much wider than for the run-up gauges, and appear to demonstrate a different trend from that identified above. No further quantitative analysis of these particular results have therefore been possible.
Run-up measurements on the stabit armoured slopes using run-up gauges were not available, so efforts were concentrated on analysing the video recordings. Run-up levels derived from these by fitting the Rayleigh probability distribution are shown in Fig 4.9. Again the data values are widely scattered. They do, however, appear to follow the general trends shown in Figs 4.5 and 4.6, but at lower values, closer to those shown in Figs 4.7 and 4.8. The scatter of the results did not justify fitting equations of the form proposed by Losada and Gimenez-Curto, as experience had shown that convergence of the iterative method used was very slow as the scatter of the data values increased.

Measurements on the diode armoured slopes were made with the run-up gauges, and levels derived from them are shown in Fig 4.10. The levels measured are very similar to those measured on the tetrapod slopes, but appear to demonstrate a less exponential trend. It may be that this is principally due to the limited number of slope angles selected, 1:1.33 and 1:1.50 only. Simple linear regression was therefore used to give mean trend lines valid over the range of conditions tested:

\[ R_s/H_s = 0.863 + 0.025 \text{Ir}' \]  
\[ R_2/H_s = 1.297 + 0.011 \text{Ir}' \]

Curves of the general form of equation 4.18 have also been fitted to these results:

\[ R_s/H_s = \frac{6.42 \text{Ir}'}{\text{Ir}' + 27.7} \]  
\[ R_2/H_s = \frac{3.50 \text{Ir}'}{\text{Ir}' + 8.18} \]

It should be noted, however, that the correlation calculated was relatively low, and these curves are in any case valid only over the range of conditions studied.

During the earlier study, run-up levels had also been measured on SHED armoured slopes using the video technique only, and are shown in Fig 4.11. These results appear to show higher levels of run-up on the SHED armoured slopes than were seen on the other armoured slopes. In part this might be expected as the SHED units are relatively open and are laid in a single layer only. However it should be noted that these units are much smaller in relation to the wave heights, and lengths, than the other units tested in this study. This may itself give rise to a reduced...
relative roughness, an effect that is discussed later. As, however, these tests yielded few test results, little further quantitative analysis of these results was performed than to fit equations of the form of equation 4.18:-

\[ R_s/H_s = \frac{1.33 I_r}{I_r' + 0.0002} \]  
\[ R_2/H_s = \frac{2.78 I_r}{I_r' + 1.392} \]

Again, however, the correlation calculated was relatively low.

5 THE PROBABILITY DISTRIBUTION OF RUN-UP CRESTS

5.1 Probability distributions

A number of tests were selected from the mass of results available, for detailed statistical analysis. The choice was intended to produce a representative sample of the total series of experiments. Those selected are listed below, and include roughly similar tests with only the armour material changed, and also experiments with similar units and only the wave conditions changed. In each case random waves were used, specified by a JONSWAP spectrum defined by significant wave height and mean period. Results in this chapter have been scaled by a Froudian scale of 1:32.5.

<table>
<thead>
<tr>
<th>Armour Type</th>
<th>Structure Type</th>
<th>Slope</th>
<th>Hs (m)</th>
<th>Tz (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>Smooth</td>
<td>1:1.5</td>
<td>5.2</td>
<td>9.2</td>
</tr>
<tr>
<td>Smooth</td>
<td>Smooth</td>
<td>1:2.0</td>
<td>5.2</td>
<td>9.2</td>
</tr>
<tr>
<td>Antifer Cube</td>
<td>Antifer Cube</td>
<td>1:1.5</td>
<td>4.8</td>
<td>8.8</td>
</tr>
<tr>
<td>Diode</td>
<td>Diode</td>
<td>1:1.5</td>
<td>4.8</td>
<td>8.8</td>
</tr>
<tr>
<td>Diode</td>
<td>Diode</td>
<td>1:1.5</td>
<td>2.9</td>
<td>8.1</td>
</tr>
<tr>
<td>SHED</td>
<td>SHED</td>
<td>1:1.5</td>
<td>7.0</td>
<td>11.2</td>
</tr>
<tr>
<td>SHED</td>
<td>SHED</td>
<td>1:2.0</td>
<td>7.0</td>
<td>11.2</td>
</tr>
</tbody>
</table>

It is well known that there is very little correlation between sea surface elevation and water level on a beach or structure. Consequently the number of run-up crests may be lower than the number of waves in a run. For the present purpose, the number of run-up peaks measured during a test is used in preference to the number of waves, although the latter would be easier to obtain in practice.

Various theoretical probability density functions have been fitted to run-up in the past, with different degrees of success, depending upon the conditions. In
This study, the Gamma, Rayleigh and Weibull distributions, defined below in equations 5.1-5.3, are fitted to the data collected during the seven selected tests. The results are shown in Figures 5.1-5.7.

\[ \text{Gamma pdf} \quad p(R) = \frac{b^c}{R^c} \exp \left\{ -\frac{bR'}{R} \right\} R'(c-1) \quad 5.1 \]

where for Gamma \( b = \frac{R'}{\alpha^2}, \) \( c = b R' \)

\[ \text{Rayleigh pdf} \quad p(R) = \frac{1.57}{R'^2} \exp \left\{ -0.785 \left( \frac{R'}{R} \right)^2 \right\} R' \quad 5.2 \]

(derived by substitution of \( x_{\text{rms}} = 1.129z \) into \( p(x) = (2x/x_{\text{rms}}) \exp \left\{ -[x/x_{\text{rms}}]^2 \right\} \).)

\[ \text{Weibull pdf} \quad p(R) = \frac{c}{b} \exp \left\{ -\left[ \frac{R'}{b} \right]^c \right\} \left( \frac{R'}{b} \right) (c-1) \quad 5.3 \]

where \( R' = R-a \)

and \( a \) may be taken to represent the level below which no run-up peak will be recorded.

The Gamma distribution is defined by two parameters from the data, i.e. its mean and standard deviation, whilst the Rayleigh distribution is defined by only one or the other of these values. The Weibull distribution, however, is more pliable, in that the parameters \( b \) and \( c \) can take any values that produce a good fit. They are actually obtained by a graphical method which takes into account all the data points.

The zero level of each distribution is fairly arbitrary, being the still water level at the time of installation of the recording apparatus. This may not be the minimum level of run-up crests, and so the parameter \( a \) appears in all three equations, representing the sharp lower cut-off, below which run-up peaks do not occur.

A comparison of error squared test values suggests that the Weibull distribution generally gives the best fit, which is not surprising since it has the most flexible format of the three. The experimental results do not form as smooth a curve as might be expected if any theoretical function were truly a good fit for the data. Only a few of the examples quoted would pass the usual statistical tests for agreement. However, any of the three would produce a useful guide to estimating extreme events based on all the results, rather than just a few high values.

In the seven experiments which are plotted here, the Weibull distribution produces a slightly better fit than does the Rayleigh, which in turn gives a better fit than the Gamma. It is also noted that the SHED and smooth slope results are generally better matched by the theoretical distributions than are the diode and antifer results.
5.2 Extreme levels

The lack of consistency in the present results and in those given elsewhere in the literature indicates that no single probability density function provides a good fit in all cases. For practical purposes it may be better to measure the run-up design parameters, i.e. mean, standard deviation, $R_2$ and $R_S$ directly from the original data, than to fit an arbitrarily selected probability distribution.

The prediction of extreme levels, such as $R_2$, may be made by calculating a typical run-up level, such as $R_S$, and then determining $R_2$ from that level. This has often been done by assuming that these run-up levels are linked by a Rayleigh probability distribution. For such a distribution of either wave height, or run-up level, the ratio of the 2% exceedance value to the significant value will be given by:

$$\frac{H_2}{H_s} = \frac{R_2}{R_S} = 1.399$$

For this study, the results of measurements on smooth slopes, derived by direct interpolation from the output of the run-up gauges, give a mean value of $R_2/R_S$ of 1.45, slightly higher than would be predicted by a Rayleigh distribution. This increased ratio is in general agreement with the results of work by Kamphuis and Mohamed (21) who found $R_2/H = 2.4$, rather than 2.23 which would be predicted for a Rayleigh distribution. The ratio of $R_2$ to $R_S$ has also been examined for the armoured slopes considered in this study. Values of the mean ratio, and its standard deviation are shown below:

<table>
<thead>
<tr>
<th>Slope</th>
<th>Measurement method</th>
<th>Spectral type</th>
<th>$R_2/R_S$ Mean</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>Gauge</td>
<td>J</td>
<td>1.45</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>1.45</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Video</td>
<td>J</td>
<td>1.58</td>
<td>0.16</td>
</tr>
<tr>
<td>Tetrapods</td>
<td>Gauge</td>
<td>J</td>
<td>1.36</td>
<td>0.07</td>
</tr>
<tr>
<td>Antifer</td>
<td>Gauge</td>
<td>J</td>
<td>1.36</td>
<td>0.08</td>
</tr>
<tr>
<td>cubes</td>
<td></td>
<td>M</td>
<td>1.39</td>
<td>0.11</td>
</tr>
<tr>
<td>Diodes</td>
<td>Gauge</td>
<td>J</td>
<td>1.37</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>1.41</td>
<td>0.09</td>
</tr>
</tbody>
</table>

It is interesting to note that $R_2/R_S$ for armoured slopes was always less than on smooth slopes, and generally less than would be predicted by a Rayleigh distribution. The higher values of $R_2$ produced by the...
video analysis have been discussed in Section 4.3 above. Generally it would appear that the use of the Rayleigh probability distribution to predict 2% exceedance run-up levels will be slightly conservative on armoured slopes, but may lead to a small under-prediction of level on smooth slopes.

6 DISCUSSION OF RUN-UP LEVELS MEASURED

6.1 General

The relative run-up levels measured in this study have been presented in chapter 4. Comparisons have been made of the relative effects of different incident spectral types, wave heights and periods, structure slope angles, and the different measurement techniques. The results have been described in terms of relative run-up levels and the modified Iribarren number. In this chapter the run-up levels measured are compared with those predicted by some of the methods identified by Allsop, Franco and Hawkes(8). Areas of disagreement between the results of this study and some of the predictions are explored.

6.2 Smooth slopes

The literature review(8) identified a number of prediction methods for regular wave run-up on smooth slopes, some theoretical approaches for irregular wave run-up, and the single prediction method based on irregular wave measurements proposed by Ahrens(5). One of the simplest prediction expressions for regular wave run-up identified in the review is a 3-part expression advanced by Losada and Gimenez-Corto(22) which is compared with results of this study in Fig. 6.1. Over the range of input variables considered in this study, this prediction method appears to give run-up levels lying between \( R_s \) and \( R_2 \). The results indicate a shallower trend over \( 2.5 < \text{Ir} ' < 4.0 \) than that predicted, but continue to give reducing run-up values for \( \text{Ir} ' > 4.0 \). The use of this prediction expression to estimate \( R_s / H_s \) would appear to be generally conservative.

Further curves predicting regular run-up may be calculated from an expression advanced by Chue(23). These curves, however, demonstrate a trend of increasing relative run-up levels with increasing values of \( \text{Ir} ' \), contrary to that shown by results from this study, as plotted in Fig. 6.2. Furthermore Chue's expression argues an effect of slope angle not demonstrated by these results. Other reservations as to the validity of this expression have been discussed in the literature review(8).
Other prediction methods have also been considered for regular wave run-up, and Allsop, Franco & Hawkes (8) give tables of relative run-up, R/H, for various structure slopes and sea steepnesses. The Miche and Hunt formulae have been used to calculate values of relative run-up shown in Fig 6.3, and compared with levels measured in this study, as described by equations (4.6) and (4.9). This comparison shows that the use of the Hunt and Miche equations introduces an effect of structure slope not seen in the measured results. The values of R/H computed from these equations are however relatively close to the measured values of R/H described by equation (4.6).

Less good agreement is however shown by values of R/H derived from the Shore Protection Manual (24) (SPM Fig 7.12) and presented in Fig 6.4. These show an increasing difference from values measured in this study at the higher values of Ir.

For irregular wave run-up, Ahrens (5) has suggested a general expression for smooth slopes that may be written:

\[ \frac{R}{H_o} = C_1 + C_2 \left( \frac{H_s}{g T_p^2} \right) + C_3 \left( \frac{H_s}{e T_p^2} \right)^2 \]  

where \( R \) represents \( R_1 \), \( R_2 \), or \( R \), and \( C_1 \), \( C_2 \), and \( C_3 \) are empirical coefficients determined by regression analysis of the data considered. Ahrens gives tables of values of \( C_1 \), \( C_2 \), and \( C_3 \) for each of \( R_1 \), \( R_2 \), and \( R \) for a range of slopes. Values of relative run-up \( R_1/H_s \) and \( R_2/H_s \) calculated using this method for slopes of cot \( \alpha = 1.5 \) and 2.0 are compared in Fig 6.5 with results of this study. At significant run-up level, \( R \), those values predicted by Ahrens lie well above those measured in this study. The reason for these discrepancies are not clear. It may be noted that values of \( R_1/H_s \) measured in this study are much closer to values of R/H predicted by Hunt/Miche, than are those predicted by Ahrens' method. The opposite is however true of the results shown in Fig 6.4 calculated from the Shore Protection Manual (24). This discrepancy of values of relative run-up on smooth slopes was not identified during the model tests, it has therefore not been possible to resolve the reasons for these differences during the course of this study.

It would appear however, that the data from which Ahrens' prediction method is derived shows an effect of structure slope angle on the run-up levels measured that was not seen in this study, see Fig 4.4. A comparison of the relative run-up \( R_1/H_s \) for a 1:1 slope given by Ahrens with the regression line for \( R_1/H_s \), equation (4.6), for all slopes measured in this study shows much better agreement. Ahrens' results on
shallower slopes, however, give higher values of relative run-up. The extent of this discrepancy is of the order of 30%.

6.3 Armoured slopes, comparison with predictions

Virtually no measurements have been made of run-up on armoured slopes under random wave attack, so prediction methods are generally based on the results of tests on armoured slopes with regular waves, or on smooth slopes with random waves. The use of a simple roughness factor to estimate run-up levels on armoured slopes from those predicted for the equivalent smooth slopes has been discussed previously by Losada & Gimenez-Curto(6) and by Allsop, Franco & Hawkes(8), both of whom conclude that such a method is generally unsatisfactory. This may be further demonstrated by contrasting the run-up behaviour of smooth and tetrapod armoured slopes as presented in Figs 4.1 and 4.5 respectively. In this section attention is therefore directed principally to those prediction methods based on regular wave tests on armoured slopes. Many such test results have been analysed by Losada & Gimenez-Curto(6) who have fitted equations of the general form:

\[ \frac{R}{H} = A \left[ 1 - \exp\left( B \frac{I_r}{r} \right) \right] \]  

(6.2)

Another general expression is that ascribed to CERC by Günbak(20), that has already been given as equation (4.18):

\[ \frac{R}{H} = \frac{A I_r}{I_r + B} \]

Values of these different empirical coefficients are presented in both references 6 and 20, and are reproduced here as equations (6.3)-(6.11); and values for equation (4.18) may also be derived from expressions quoted by Seelig(27):
<table>
<thead>
<tr>
<th>Armour unit</th>
<th>General eq number</th>
<th>Coefficients</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrapod</td>
<td>(6.2)</td>
<td>0.934, -0.750</td>
<td>(6.3)</td>
</tr>
<tr>
<td>Quadripod</td>
<td>(6.2)</td>
<td>1.538, -0.248</td>
<td>(6.4)</td>
</tr>
<tr>
<td>Dolos</td>
<td>(6.2)</td>
<td>1.216, -0.568</td>
<td>(6.5)</td>
</tr>
<tr>
<td>Rock (a)</td>
<td>(6.2)</td>
<td>1.370, -0.596</td>
<td>(6.6)</td>
</tr>
<tr>
<td>Rock (b)</td>
<td>(4.18)</td>
<td>1.6, 2.0</td>
<td>(6.9)</td>
</tr>
<tr>
<td>Rock (c)</td>
<td>(4.18)</td>
<td>1.37, 1.98</td>
<td>(6.10)</td>
</tr>
<tr>
<td>Rock (d)</td>
<td>(4.18)</td>
<td>2.40, 2.51</td>
<td>(6.11)</td>
</tr>
<tr>
<td>Rip-rap (a)</td>
<td>(6.2)</td>
<td>1.789, -0.455</td>
<td>(6.7)</td>
</tr>
<tr>
<td>Rip-rap (b)</td>
<td>(6.2)</td>
<td>1.451, -0.523</td>
<td>(6.8)</td>
</tr>
</tbody>
</table>

Prediction equations (6.3) and (6.4) for regular wave run-up on tetrapod and quadripod armoured slopes are shown in Fig 6.6, together with results for the tetrapod armoured slopes from this study. There is very good agreement between the significant run-up levels measured in this study, as given by equation (4.14) and the regular wave run-up levels measured on the quadripod slopes, as given by equation (6.4), but less good with that fitted to the tetrapod slopes, equation (6.3). However, considering both regular wave data sets together, the prediction methods show very good agreement with results measured in this study.

No run-up measurements on antifer cubes have been presented elsewhere in the literature, so it has not been possible to offer any direct comparisons with the results of this study as given in equations (4.16) and (4.17) in Fig 6.7. Run-up on rock armoured slopes has however been described by equations (6.6) and (6.9), and on rip-rap slopes by equations (6.7) and (6.8). These different expressions for regular wave run-up on rock armoured slopes are shown in Fig 6.8. As expected the more porous slopes permit lower run-up levels than the rip-rap slopes. The trend for \( R_s/H_s \) for antifer cubes given by equation (4.16) lies slightly below those for \( R/H \) on rock slopes given by equations (6.6) and (6.9).
It appears from the results of run-up measurements on armoured slopes that the run-up performance of such slopes depends upon the relative size of the armour unit with respect to the incident waves. This dependency is touched upon briefly by Stoate(23), Ahrens(5) and Gadd et al(26), all of whom generally consider waves of heights approaching the stability limit of the particular armour units concerned. Stability considerations will set a lower limit to the armour unit size for a particular site and set of wave conditions. The upper limit will usually be set by economic considerations and will often simply be determined by applying a suitable safety factor to the size determined for stability, although an economic analysis may, in some circumstances, suggest larger units may be more appropriate. Recently, however, the high relative stability of single layer, regularly placed units such as Cobs, SHEDS or Diodes, had led to the use of armour units much smaller in relation to the incident wave height than hitherto. As the relative size decreases, so the thickness of both armour and underlayer over which wave energy may be absorbed is also reduced, and run-up levels may therefore increase. In the limit the run-up on such a slope will approach that for a smooth slope.

Unfortunately the influence of relative armour unit size on the run-up performance of an armoured slope was identified as of some significance only as a result of analysis of the test measurements. It was not therefore possible to quantify the effect within this study.

A number of qualitative conclusions may however be drawn from the results of these tests. As might be expected, any reduction in relative run-up is greatest at the lower values of Ir', generally corresponding to the shallower structure slopes, hence having the greater slope distances over which the roughness and porosity of the armour may act. Conversely, at large values of Ir', again generally corresponding to steeper structure slopes, waves will tend to surge up and down the slope experiencing relatively less reduction in run-up levels due to the armour roughness and porosity. In the limit run-up levels might be expected to tend to those for standing waves against a vertical wall. Theoretical expressions for such circumstances are discussed by Allsop, Franco & Hawkes(4), and relative run-up is shown to depend on water depth, wave height and/or length. For waves of a mean steepness, H/Lo, of about 0.04, the run-up on a vertical wall in deep water may be estimated as around R/H = 1.2. It should be noted that higher values of relative run-up will result from less steep waves and/or shallower water.
From run-up measurements, different armour units have been shown to give different relative run-up curves. Within this study it has not been possible to identify the effect on run-up of armour unit size or of armour layer porosity for each armour unit. It may however be reasonably expected that curves of significant run-up levels, \( R_s/H_s \), are bound at the upper limit by the curves for smooth slopes, and at the lower limit by curves tending towards the vertical wall limit, say around \( R/H = 1.2 \) for the example considered above. The lowest reliable run-up levels in this study were measured on the Diode armoured slope.

Within the range of structure slopes and wave conditions considered, the effect of wave steepness, \( H_s/L \), structure slope, \( \tan \alpha \), and wave height, \( H_s \), on run-up levels \( R_s \) and \( R_2 \) are well described by suitably chosen expressions for relative run-up, \( R_s/H_s \) and \( R_2/H_s \) in terms of the modified Irribarren number, \( I_r' \).

Both JONSWAP and Moskowitz spectral shapes were used in testing, and no significant effects on run-up behaviour due to spectral type were noted.

Measurements on smooth slopes gave values of relative run-up, both at 2\% exceedance, \( R_2 \), and significant, \( R_s \), levels that are markedly lower than those predicted by Ahrens' method\(^5\). The reason for this discrepancy has not been identified, and further work will be necessary to resolve the apparent differences. It is noted however that the significant run-up levels, \( R_s \), measured in this study compare reasonably closely with the regular wave run-up levels, \( R \), predicted by Hunt and Miche and by Losada & Gimenez-Curto\(^22\).

Random wave model tests on both smooth and armoured slopes have confirmed the conclusions of the literature review\(^8\), and of other reviewers\(^6\), that the use of a simple roughness coefficient to estimate run-up levels on armoured slopes from those predicted on smooth slopes is likely to give under-predictions of run-up levels in some circumstances and over-predictions in others.

The run-up performance of rough armoured slopes may generally be described well by an expression of the exponential form of equation (6.2) or the reciprocal form of equation (4.18). Both these expressions require the determination of values of the empirical coefficients. From the limited comparisons possible it appear that there is good agreement between significant run-up levels measured in this study and
regular wave run-up levels analysed previously. The results of some of those regular wave tests might therefore be used to estimate significant run-up levels. The limitations of the original regular wave tests must however be appreciated.

It is clear from this study that run-up levels on armoured slopes are dependent on the relative size of the armour unit to the waves. This dependency on relative armour unit size has been touched upon very briefly by other authors. It is however of greatest importance for the recently introduced single layer units such as the cob, the SHED and the Diode, where their very high stability to weight ratio may lead to the use of armour units of a much smaller relative size than hitherto. The scope of these model tests was not however sufficient to quantify the effect.

A number of different probability distributions have been fitted to run-up measurements on both smooth and armoured slopes. The lack of consistent agreement with any one of the probability distributions tested in this study, and with those considered elsewhere in the literature, indicates that no single probability density function is likely to provide a good description in all cases. It would therefore appear to be more practical for design purposes to predict a single run-up level such as \( R_n \), and then to estimate extreme levels such as \( R_2 \) using a Rayleigh distribution. On smooth slopes this may lead to a slight under-prediction of extreme levels, but on armoured slopes the test results indicate that such a method will generally overpredict the extreme levels by a small margin.

It should be emphasised that the run-up prediction methods considered in this study should not be used to estimate overtopping discharges for seawalls. If significant overtopping is anticipated, the design method outlined by Owen\(^{(3)}\) should be used where appropriate, supported if necessary by hydraulic model tests.

### 8 ACKNOWLEDGEMENTS

This report summarises the results of model tests and analysis conducted by members of the Maritime Engineering Department of Hydraulics Research. They were assisted by L Franco on study leave from the University of Rome.

The model tests were designed and conducted by F A Jackson and N W H Allsop assisted by L Franco and A P Bradbury. The analysis was conducted by N W H Allsop, F A Jackson, P J Hawkes and L Franco. The report was written by N W H Allsop.
REFERENCES


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- Measured data
- Gamma dist
- Weibull dist
- Rayleigh dist

$H_s = 6.99$ m, $T_z = 11.2$ s
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