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SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS

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ABSTRACT

Sediment movement caused by waves and currents and the interaction of the two is one of the more salient features of estuaries and coasts. The undertaking of engineering works in such areas, for example the construction of breakwaters and docks or the dredging of shipping channels may radically affect this movement of sediment and such engineering works may also be significantly affected by sediment movement. To enable predictions to be made of the impact of engineering works there is an interest in the development of numerical models to predict sediment movement in estuary and coastal situations. Since both waves and currents may be significant in these areas the models must include the effects of both. A major element of such a model is a theory for sediment transport under waves and currents. A number of sediment transport theories have been proposed but there has been little work done comparing the different theories and investigating their behaviour on observed data. In this study a number of sediment transport theories are compared with field and flume data. Their performance is described and suggestions made for improvements. In the theories considered one of the variables used to determine the sediment transport rate is the bed shear stress. The study revealed that the predictions of the various theories are sensitive to the expression used for the bed shear stress developed under waves and currents and so such expressions are discussed. The work should aid in the development of numerical models to predict the behaviour of sediment in estuary and coastal situations under the action of both waves and currents.

CONTENTS

FIGURES (cont'd)

1 INTRODUCTION

Sediment movement caused by waves and currents and the interaction of the two is one of the more salient features of estuaries and coasts. The undertaking of engineering works in such areas, for example the construction of breakwaters and docks or the dredging of shipping channels may radically affect this movement of sediment and such engineering works may also be significantly affected by sediment movement. To enable predictions to be made of the impact of engineering works there is an interest in the development of numerical models to predict sediment movement in estuary and coastal situations. A major element of such a model is the equations which describe the movement of sediment. Though in estuaries the primary agent for moving sediments is currents the presence of waves may have a significant effect. Meanwhile in coastal situations, though the action of waves may predominate, the effect of currents may not be negligible. Thus, in order to use a numerical model to make predictions of sediment transport in these situations it is necessary to use a theory of sediment transport which allows for the effect of both waves and currents.

A large amount of effort has been expended on studying the mechanisms of sediment transport and developing theories which will predict it. This effort has been chiefly directed at studying sediment transport under uni-directional flow as exemplified by sediment movement in rivers. In an estuary or on a coast, however, sediment is moved by a combination of both waves and currents. In this report a number of theories of sediment transport under waves and currents are considered and their predictions are compared with observed sediment transport rates. Since the object of the study was to consider which sediment transport theories were suitable for inclusion in appropriate computational models only a selection of the available theories were considered.

2 BED SHEAR-STRESS DUE TO WAVES AND CURRENTS

All the theories of sediment transport considered use some form of expression for the bed shear-stress developed under waves and currents. Since the sediment transport is sensitive to this the expression used for the bed shear-stress may have a significant effect on the performance of the sediment transport theory considered. We, therefore, now give a brief review of different expressions for the bed shear stress under waves and currents, τ_{wc}. They are all based on corrections to the shear stress under currents alone, denoted by τ_c. Such an approach may be valid, provided that the effect of the waves is small in comparison with the currents but the equations become increasingly suspect as the significance of the waves increases.

2.1 Bijker and Swart

Bijker assumed that the fluid motion under waves and currents could be regarded as the superposition of a uni-directional turbulent boundary layer and a linear, first-order, inviscid wave. From the resulting velocities Bijker calculated the instantaneous shear under waves and currents. This was then integrated over a wave period to give a mean bed shear. If the absolute value of the shear is taken as the integrand one obtains, for waves and currents in the same direction:

$$\frac{\tau_{wc}}{\tau_c} = 1 + \tfrac{1}{2} \left(\xi \frac{u_o}{V}\right)^2 \qquad (1)$$

where

$$\xi = p_B \frac{\kappa C}{g^{\frac{1}{2}}} \qquad (2)$$

κ Van Karman's constant, C Chezy coefficient, P_B is a constant taken by Bijker to be 0.45, Uo is orbital velocity, V current velocity (Bijker 1967, p 49).

Swart (1976) uses equation (1) to determine the bed shear but in the definition of ξ he replaces the constant P_B by a variable dependent upon the wave amplitude and the roughness of the bed. The equation (2) for ξ is then replaced by

$$\xi = C \left(\frac{f_w}{2g}\right)^{\frac{1}{2}} \qquad (3)$$

where f_w is the Jonsson wave friction factor (Jonsson, 1966).

Equations (1) and (3) seem to be those used in all the sediment transport equations modified by Swart.

Bijker, however, points out that in deriving equation (1) the absolute value of the shear was used and that if the direction, as well as the magnitude of the shear, is taken into account the mean over a wave period becomes:

$$\frac{\tau_{wc}}{\tau_c} = A \int\Big[\left(1 + \xi \frac{u_o}{V} \sin \omega t \sin \phi\right) \sqrt{\left(1 + \xi^2 \frac{u_o^2}{V^2} \sin^2 \omega t + 2 \xi \frac{u_o}{V} \sin \omega t \sin \phi\right)}\Big] dt \qquad (4)$$

where ϕ the angle between the wave direction and the normal to the current (Bijker, 1967, p 35). Over a limited range of $\xi \frac{u_o}{V}$, Bijker evaluates this elliptic integral and approximates the value by a function of the form

$$\frac{\tau_{wc}}{\tau_c} = a + b \left(\xi \frac{u_o}{V}\right)^c \qquad (5)$$

for value of $\xi \frac{u_o}{V}$ in the range of $0.4 < \xi \frac{u_o}{V} < 10$. Bijker shows that the expression (1) is a successful approximation to (5) provided $\xi \frac{u_o}{V} < 1$ i.e. when the current is sufficiently large that the shear does not reverse, but it is apparent that it rapidly becomes a poor approximation for $\xi \frac{u_o}{V} > 1$ (e.g. the values of $(\tau_{wc}/\tau_c - 1)$ from equations (1) and (5) differ by 100% for $\xi u_o/V > 5$, see Table 1). The conclusion must be that equation (1) and any method which utilises it is not reliable for values of $\xi \frac{u_o}{V}$ greater than 1.

2.2 Willis (1978)

Willis assumes that the shear is proportional to the square of the instantaneous velocity where the instantaneous velocity is given by

$$V_{TOT}^2 = \left(V + u_0 \sin \frac{2\pi t}{T} \cos \alpha\right)^2 + \left(u_0 \sin \frac{2\pi t}{T} \sin \alpha\right)^2 \qquad (6)$$

where α is the angle between the currents and the wave directions. Averaging V_{TOT}^2 over a wave period one obtains

$$\overline{V_{TOT}^2} = V^2 + \frac{u_o^2}{2} \qquad (7)$$

Willis then assumes that the uni-directional Chezy friction factor can be applied to the uni-directional flow term and that the Jonsson wave friction factor can be applied to the oscillatory term so that the resulting shear is

$$\tau_{wc} = \rho \left(\frac{V^2}{C^2} + \frac{f_w}{4} u_o^2\right) \qquad (8)$$

In taking the absolute value of V_{TOT} Willis ignores the reversal in direction of the shear that will take place if $u_0 \cos\alpha$ is greater than V. By using the Jonsson friction factor the expression for the shear is for the maximum shear developed. This suggests that when the maximum orbital velocity of the wave motion exceeds the current velocity the Willis expression will overpredict the bed shear in a manner similar to Swart's approach. This can be substantiated by the following analysis.

Following Willis's assumption we can suppose that

$$\tau_c = \rho \frac{V^2}{C^2} \qquad (9)$$

then from equation (8) we have

$$\frac{\tau_{wc}}{\tau_c} = \rho \frac{\left(\frac{V^2}{C^2} + \frac{f_w}{4} u_o^2\right)}{\rho \frac{V^2}{C^2}} \qquad (10)$$

$$= 1 + \frac{C^2 f_w}{2g} \times \frac{2g}{4} \frac{u_o^2}{V^2} \qquad (11)$$

$$= 1 + \frac{2g}{4} \left(\xi \frac{u_o}{V}\right)^2 \qquad (12)$$

which is comparable with equation (1)

Willis inserts an empirical constant so that equation (12) becomes

$$\frac{\tau_{wc}}{\tau_c} = 1 + 0.6 \frac{2g}{4} \left(\xi \frac{u_o}{V}\right)^2$$

2.3 Van de Graaff and van Overeem (1979)

Graaff and Overeem use Bijker's approximation, of the form

$$\frac{\tau_{wt}}{\tau_c} = a + b \left(\xi \frac{u_o}{V}\right)^C \qquad (5)$$

to the elliptic integral in equation (4). They use the approximation quoted by Bijker of

$$\frac{\tau_{wc}}{\tau_c} = 0.75 + 0.45 \left(\xi \frac{u_o}{V}\right)^{1.13} \text{ for } = 0 < \theta < 20° \qquad (13)$$

For the difference between this equation and equation (1) see Table 1. Unfortunately in the case of zero waves, i.e. $u_o = 0$, $\tau_{wc} \neq \tau_c$

Computed values of $\frac{\tau_{wc}}{\tau_c}$ are shown in Fig 1, taken from Bijker (1967), which suggests that any approximation of the form $a + b \left(\xi \frac{u_o}{V}\right)^c$ will be inaccurate over a large range of values of $\xi \frac{u_o}{V}$.

It is clear from the above analysis that the equations used by Swart and Willis for shear stress under waves and currents are only satisfactory approximations to Bijker's equations when the wave motion is small relative to the current ($\xi u_0/V < 2$) and that outside this range the predictions of the equations using these expressions for the shear stress are likely to be unsatisfactory. This is confirmed by the available data. It further suggests that the predictions of Van de Graaff and van Overeen should be an improvement on those of Swart and Willis in the range $2 < \xi u_0/V < 10$ but that problems may arise for larger values of $\xi u_0/V$. This is partially confirmed by available data.

The whole approach of Bijker-Swart must be re-appraised in the situation where the wave motion is large relative to the current. In this case regarding the total shear as that due to currents with a correction factor to account for the waves seems to be fraught with difficulties. Swart (1974) did propose an expression for (τ_w/τ_{wc}) but it is difficult to know how this can be interpreted in the context of sediment transport under waves and currents.

The various approximations have been compared with the expression given in equation (4) derived by Bijker but at present there are no indications as to how reliable this equation is. It cannot be over stressed that the adequate prediction of sediment transport rates is unlikely without an accurate expression for the bed shear stress.

A number of studies have been made into the flow structure under waves and currents (Lundgren, 1972; Grant and Madsen, 1979 and Bakker and Doorn, 1978) from which the shear stress developed on the bed may be determined. More recent work by Fredsoe (1983) and IOS (private communication) have provided a greater insight into both the flow structure and the resulting shear stress. The models, however, that have been developed to determine shear stress are such that they are, at present, too complicated to include in large, two-dimensional numerical models in which such a calculation must be repeated many thousands of times but it is hoped that the insight provided by such models may lead to simple expressions for shear stress which are more accurate than those presently employed.

3 THEORIES OF SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS

Before considering any specific theories of sediment transport under waves and currents in detail, a few general observations will be made about such theories. A large amount of information has accumulated on sediment transport under uniform currents and it is an obvious requirement for any theory involving both waves and currents that, in the limit as the wave height tends to zero, the theory must be compatible with such information. If there were a systematic explanation of sediment transport under waves alone then a further requirement would be that in the limit as the current tends to zero, the theory must be compatible with it.

The first attempt to derive a theory for sediment transport under waves and currents based on local values for flow and waves was made by Bijker (1967) in which he adapted an existing theory due to Frijlink (1952) for sediment transport under currents alone. This approach, in which a sediment transport theory for currents alone is adapted to waves and currents has been followed by many investigators since (Swart 1976, Willis 1978 and Graaff and Overeen 1979).

In adapting a steady-state, uni-directional flow theory to a theory for waves and currents it is frequently assumed that the mechanisms of sediment transport remain the same and that the constant effective shear or shear velocity in the currents only case may be replaced by the value developed by the combination of waves and currents. While this procedure can be criticised on a number of counts it may be effective in cases where the effect of the currents dominates that of the waves. One would expect, however, since the flow structure under waves is entirely different from that under currents that problems might occur in those cases where the waves and currents are comparable or where the effect of the waves exceeds that of the currents.

A brief description follows of the various sediment transport theories compared in this study. The criterion used for selection was that they should all predict local transport rates in terms of local values of the relevant variables and that they should all be sufficiently simple to incorporate in large numerical models. This excluded immediately theories which calculate global transport rates in terms of offshore conditions alone and some of the complicated theories which involve solving differential equations throughout the depth to determine the sediment concentration profile.

In problems with simpler geometries, like infill of a dredged channel, the simpler geometry implies that the sediment transport theory has to be evaluated at fewer points. Thus it is practicable to expend more computational effort on any one sediment transport evaluation and so it is possible to include more complicated sediment transport theories than the ones considered in this report.

3.1 Frijlink-Bijker (Bijker 1967)

The Frijlink (1952) sediment transport equation may be written as:

$$S = 5V \frac{(\mu g\ D_{50})^{\frac{1}{2}}}{C} \exp\left(\frac{-0.27(s-1)D_{50}g}{\mu V_*^{\ 2}}\right) \qquad (14)$$

$$\text{where } \mu = \left(\frac{C}{C_{D_{90}}}\right)^{3/2} \qquad (15)$$

$$\text{and } C_{D_{90}} = 18 \log\left(\frac{12d}{D_{90}}\right) \qquad (16)$$

Bijker proposed replacing V_* by V_{*wc} and replacing the constant 5 by a constant appropriate to waves and currents. Using the Bijker data, Swart assigned the value of 3.66.

3.2 Engelund & Hansen - Swart (Swart, 1976)

Engelund and Hansen (1967) developed a sediment transport equation which may be expressed as

$$G = \frac{0.05 \rho_s g^{\frac{1}{2}} V^2 D_{50}^{\frac{1}{2}}}{(s-1)^{\frac{1}{2}}} \left[\frac{V_*^{\ 2}}{gD_{50}(s-1)}\right]^{3/2} \qquad (17)$$

$$= \frac{0.05 \rho_s}{g(s-1)^2 D_{50}} V^2 V_*^{\ 3} \qquad (18)$$

Swart expresses the V^2 term as $Vx(\frac{CV_*}{\sqrt{g}})$ using the Chezy formula to give

$$G = \frac{0.05 \rho_s}{g^{3/2}(s-1)^2 D_{50}} VC\ V_*^{\ 4} \qquad (19)$$

To take account of the effect of currents and waves he replaces the shear velocity by the shear velocity under waves and currents so that the expression for the transport rate becomes

$$G = \frac{0.05 \rho_s}{g^{3/2}(s-1)^2 D_{50}} VC(V_{*wc})^4 \qquad (20)$$

It should be noted that if the above processes are commuted, i.e. replace V_* by V_{*wc} and then replace V by $C\frac{V_*}{\sqrt{g}}$, a different equation results

$$G = \frac{0.05\,\rho_s}{g^{3/2}(s-1)^2 D_{50}} V(CV_*)(V_{*wc})^3 \qquad (21)$$

Swart uses the following expression for V_{*wc},

$$V_{*wc} = V_*(1+\tfrac{1}{2}(\xi \frac{u_o}{V})^2)^{\frac{1}{2}} \qquad (22)$$

$$\text{where } \xi = C\,(\frac{f_w}{2g})^{\frac{1}{2}} \qquad (23)$$

$$\text{and } C = 18 \log \frac{12d}{r} \qquad (24)$$

where the Jonsson friction factor f_w is given by

$$f_w = \exp\,(-5.98 + 5.21\,(\frac{a_o}{r})^{-0.19}),\ \frac{a_o}{r} > 1.57 \qquad (25)$$
$$= 0.30,\ \frac{a_o}{r} < 1.57$$

We can, therefore, see from equation (3) that the effect of the waves is to increase the transport rate by a factor $[1+\frac{1}{2}(\xi\, u_o/V)^2]^2$. If equation (5) were adopted the factor would be $[1+\frac{1}{2}(\xi \frac{u_o}{V})^2]^{3/2}$.

3.3 Ackers and White - Swart (Swart, 1976)

The Ackers and White equations (Ackers and White, 1973) for sediment transport utilise four parameters, n, A,m and C which depend upon the dimensionless grain size D_{gr}. Swart keeps these unchanged when applying the equation to sediment transport under waves and currents. The mobility in the case of currents alone is defined by

$$F_{gr} = \frac{V_*^n}{\sqrt{gD(s-1)}} \left[\frac{V}{\sqrt{32}\,\log_{10}\frac{10d}{D}}\right]^{1-n} \qquad (26)$$

and Swart defines the corresponding variable for waves and currents, F_{gr}^{wc} by

$$F_{gr}^{wc} = F_{gr} \times (\frac{V_{*wc}}{V_{*c}})^n \qquad (27)$$

The equation for sediment concentration in the case of currents alone is:

$$X = G_{gr} \frac{sD}{d} (\frac{V}{V_*})^n \qquad (28)$$

In the case of currents and waves Swart replaces this by

$$X = G_{gr} \frac{sD}{d} \left(\frac{V}{V_{*wc}}\right)^n \tag{29}$$

Thus, denoting $\frac{V_{*wc}}{V_{*c}}$ by β the equation for the sediment concentration becomes

$$X = C \left(\frac{F_{gr}}{A} \beta^n - 1\right)^m \frac{sD}{d} \left(\frac{V}{V_{*c}}\right)^n \frac{1}{\beta^n} \tag{30}$$

3.4 Ackers & White - Van de Graaff & Van Overeem (Graaff & Overeem, 1979)

Van de Graaff and Van Overeem considered that Swart's adaption was incomplete since it took no account of the effect of the orbital motion on the actual shear stress on the sediment grains. They, therefore, proposed that the effect of the waves of increasing V should be included, assuming a flat bed with bed material diameter as roughness elements. Thus equation (27) becomes

$$F_{gr}^{wc} = F_{gr}\left(\frac{V_{*wc}}{V_{*c}}\right) \times \left[V\left(\frac{1+\frac{1}{2}\left(\xi^1 \frac{u_o}{V}\right)^2}{V}\right)\right]^{1-n} \tag{31}$$

where $\xi^1 = C_D \left(\frac{f_w}{2g}\right)^{\frac{1}{2}}$ (32)

and $C_D = 18 \log \frac{10d}{D_{35}}$ (33)

and equation (29) becomes

$$X = G_{gr} \frac{sD}{d} \left(\frac{V}{V_{*wc}}\right)^n \left[V\left(\frac{1+\frac{1}{2}\left(\xi^1 \frac{u_o}{V}\right)^2}{V}\right)\right]^n \tag{34}$$

3.5 Ackers & White - Willis (Willis, 1978)

Willis adapted the Ackers and White (Ackers and White, 1973) sediment transport formula in a spirit approaching that of van de Graaff and van Overeem but differing in the details. Willis expressed the Ackers and White equations in the form

$$F_{gr} = \frac{V_{*fg}^{\,n} \; V_{*cg}^{\,1-n}}{\sqrt{gD(s-1)}} \tag{35}$$

and $X = C \left(\frac{F_{gr}}{A} - 1\right)^m \frac{sD}{d} \left(\frac{\rho^{\frac{1}{2}} P_{fg}}{\tau_{fg}^{3/2}}\right)^n \left(\frac{P_{cg}}{\tau_{cg} V}\right)^{1-n}$ (36)

where P is the power per unit area available to move sediment. In the case of waves and currents Willis defines Chezy coefficients by

$$C_{cg} = 5.75 \log_{10} \frac{11d}{D} \quad (37)$$

and $$C_{fg} = 5.75 \log_{10} \frac{11d}{r} \quad (38)$$

where r is the roughness of the bed.

The combined shear under waves and currents is then defined by

$$\tau_{cg} = \rho \left(\frac{V^2}{C_{cg}^2} + \frac{f_w}{4} u_o{}^2\right) \quad (39)$$

$$\tau_{fg} = \rho \left(\frac{V^2}{C_{cg}^2} + \frac{f_w}{4} u_o{}^2\right) \quad (40)$$

and the power per unit area by

$$P_{cg} = \rho \left(\frac{V^2}{C_{cg}^2} V + C_g \frac{f_w}{4} u_o{}^2\right) \quad (41)$$

and $$P_{fg} = \rho \left(\frac{V^2}{C_{fg}^2} V + C_g \frac{f_w}{4} u_o{}^2\right) \quad (42)$$

where C_g is the group velocity. Willis inserts an empirical constant $W_c{}^2$ in equations (39) to (42) so that they become (dropping subscripts)

$$\tau = \rho \left(\frac{V^2}{C^2} + W_c^2 \frac{f_w}{4} u_o{}^2\right) \quad (43)$$

$$P = \rho \left(\frac{V^2}{C^2} V + W_c^2 C_g \frac{f_w}{4} u_o{}^2\right) \quad (43)$$

c.f. W_c with constant 0.45 introduced by Bijker.

Willis found a value of W_c by fitting calculated results to observed data. He concluded $W_c = \sqrt{0.6}$

4 SEDIMENT TRANSPORT UNDER WAVES

In the limit as the current tends to zero a theory for sediment transport under waves and currents should tend to a theory for sediment transport under waves. The possibility was thus contemplated that the predictions of a theory for sediment transport under waves and currents could be improved in the wave dominated region if it was made to asymptote to a theory for movement under waves alone. It was therefore decided to look at theories for predicting sediment transport under waves to find the most suitable to act as an asymptote as the currents tend to zero.

4.1 Rance equation

Rance (HRS, 1971) based an equation for sediment transport under waves alone on the results of an investigation carried out jointly by the Hydraulics Research Station and the Coastal Engineering Research Center of Washington. In these experiments the transport of 0.2mm sand was measured under a range of wave conditions. The range of wave periods was from 4s to 11s and the wave heights were between 0.75m and 1.7m in a depth of water of 4.6m. From the results the following equation for sediment transport was derived:

$$Q_w = \frac{3H^6}{h^2 T^5} \left(\frac{1}{\sinh kh}\right)^6 \text{ lb/ft/s} \tag{44}$$

where Q_w sediment transport in lb/ft width/s
H wave height in feet
h water depth in feet
T wave period in s
k wave number $= \frac{2\pi}{L}$ where L = wave length

4.2 Sleath equations

Sleath put forward two equations for sediment transport under waves. The first equation (Sleath 1978) was fitted to oscillating tray data using 1.89mm sand, 4.24mm gravel and 3.04mm nylon pellets, and is given by:

$$\frac{Q_s}{wD^2} = 47\ (\psi - \psi_c)^{3/2} \tag{45}$$

where Q_s is the mean volume of sediment transported per unit width of bed in unit time averaged over a half cycle, w is the angular frequency, D is the equivalent sphere diameter of the sediment, ψ is a modified shields function for unsteady flow and ψ_c is the critical value of ψ for initial motion (Madsen & Grant 1976).

Sleath used the following expression for ψ:

$$\psi = \frac{f_1\ \tfrac{1}{2}\rho u_o^{\,2}}{(\rho_s - \rho)gD} \tag{46}$$

where uo is the orbital amplitude just above the bed and f_1 is a modified friction factor evaluated from

$$\frac{f_1}{f_w} = f\left(\frac{w_o}{(w\,\nu)^{\frac{1}{2}}}\ f_w\right) \tag{47}$$

which was derived from observations (Sleath 1978). f_w is Jonsson's friction (Jonsson 1965).

Subsequently Sleath (1982) derived the following equation from observations of 0.2 and 0.41mm sand

$$\frac{Q_s}{\left(\frac{\rho_s - \rho}{\rho} g D^3\right)^{\frac{1}{2}}} = 1.95 \; (\psi - \psi_c)^{3/2} \qquad (48)$$

The various methods were tested on the limited flume data available on sediment transport under waves alone, see Table 2. The Rance equation was not tested on the lightweight pumice data since the empirical Rance equation was developed from data for sand only and so the effect of sediment density is not included in the equation. The data used has severe limitations and the results are such that no firm conclusions can be drawn. It was decided to use the Rance equation for further work described later.

5 COMBINING THEORIES FOR WAVES AND WAVES AND CURRENTS

We now return to the comments made earlier that a theory for sediment transport under waves and currents should tend to a theory for sediment transport under currents as the waves tend to zero and a theory for movement under waves as the current tends to zero. Because of the derivation of the theories that we have considered so far they all tend to some recognised equation for sediment transport under currents as the waves tend to zero. The results indicate, however, that the behaviour of such theories as the currents tend to zero is completely wrong. It was, therefore, decided to look at the performance of one of the theories for sediment transport under waves and currents when it was constrained so that it would asymptote to an equation for transport under waves alone as the current amplitude tended to zero. The Ackers-White Swart equation and the Rance equation were selected. Each theory was evaluated on the appropriate data and then the values of the sediment transport were multiplied by a weighting factor and the results added. The weighting factor for the Ackers-White Swart equation was $1/(1+u_0/V)$ so that for no waves it had the value 1 and for no currents it was 0. The weighting factor for the Rance expression was $1-1/(1+u_0/V)$.

The justification for such a procedure is tenuous in the extreme. The only positive claim that can be made is that the resulting expression for sediment transport has the correct type of behaviour under currents alone and under waves alone. There was, however, a marked improvement in the predictions in the wave dominated area see Figure 18.

6 FIELD AND FLUME DATA

Field and flume data on which to compare the various theories was collected from a number of sources.

6.1 Boscombe Pier Data

Field measurements were taken by the Hydraulics Research Station at Boscombe Pier, 2km east of Bournemouth Pier, between October 1977 and February 1979. Part of the available data set has been analysed in detail (Hydraulics Research Station, 1981) and was used in this study.

The data exhibited the following range of parameters

$0.14mm < D_{50} < 0.3mm$
$3.5m < d < 5.3m$
$0.34m < H < 1.1m$
$6.1s < T < 9s$
$0.04m/s < V < 1.5m/s$
$0.26 < u_o/v < 25$

6.2 Inman and Bowen

Inman and Bowen (1962) performed laboratory measurements with the following range of parameters

$D_{50} = 0.2mm$
$0.49m < d < 0.51m$
$0.15m < H < 0.17m$
$1.4s < T < 2.0s$
$0m/s < V < 0.06m/s$
$3.6 < u_o/V < 15$

6.3 Bijker (1967)

Bijker performed laboratory experiments at Delft

$D_{50} = 0.23$
$0.14m < d < 0.38m$
$0.02m < H < 0.095m$
$0.7s < T < 2.0s$
$0.1m/s < V < 0.4m/s$
$0.14 < u_o/V < 1.54$

6.4 Vincent

Vincent performed laboratory experiments with three different sediments

Fine sand
$D_{50} = 0.23mm$
$0.4m < d < 1.0m$
$0.04m < H < 0.12m$
$0_s < T < 1.9s$
$V = 0 m/s$

Medium sand

D_{50} = 0.46mm
1.0m < d < 2.1m
H = 0.6m
1.3s < T < 2.1s
V = 0 m/s

Pumice (specific gravity 1.38)

D_{50} = 0.6mm
0.5m < d < 2.1m
H = 0.3m
0.9s < T < 2.0s
V = 0 m/s

6.5 Shibayama & Horikawa

Shibayana and Horikawa performed laboratory experiments

D_{50} = 0.7mm
0.15m < d < 0.17m
0.8m < H < 1m
0.98s < T < 1.25s
V = 0 m/s

Almost every data set used has been criticised in one way or another but none of the problems mentioned are sufficiently major to invalidate the broad conclusions drawn from the study.

7 PERFORMANCE OF SEDIMENT TRANSPORT THEORIES

The various theories described above have been tested on the data already detailed. Though the bulk of the data on transport rates showed consistent trends with the predictions of the various theories a small minority of the data exhibited anomalous behaviour with every theory. It was, therefore, decided to ignore that data for the purposes of the comparison. Figures 2 to 6 show the discrepancy ratio, that is, the ratio of the predicted to the observed transport rate, for the various theories as a function of u_o/V. As a guide to the behaviour of each theory, the mean value of the discrepancy ratio for a given sediment transport theory on a given data set was calculated and these values are given in Table 3. The parameter u_o/V represents a measure of the relative magnitude of the waves and currents, u_o/V being small when currents predominate and u_o/V being large representing predominantly waves. The Figures 2 to 6 show that the theories provide adequate predictions providing the currents dominate, that is, $u_o/V < 1$ but that if the waves predominate the theories are less satisfactory, tending to over predict the transport rates. This behaviour is not unexpected from consideration of the methods by which the theories have been derived and from the expression used for the shear stress.

In the light of the previous comments about the equations used by the various methods for the bed shear stress it was decided to replace these by equation (4). Values of the corresponding discrepancy ratios are shown in Figs 7 to 11. There is a general improvement in the predictions but the various methods still over-predict for values of $u_o/V < 1$.

The over-prediction for large values of u_o/V probably reflects the difference in transport mechanisms when waves predominate rather than currents. A soundly based predictor for this parameter range must await the development of an adequate theory for sediment transport under waves alone. In an attempt to extend the validity of the present methods various ad hoc adjustments to the effective bed shear stress were considered to see if these could improve the accuracy of the predictions. In an attempt to reduce the overprediction it was decided to reduce the bed shear stress as u_o/V increased. A selection of different weighting functions were selected on an ad hoc basis designed to reduce the bed shear stress for larger values of u_o/V. The equation for shear velocity

$$\frac{V_{*wc}}{V_{*c}} = \left[1 + \tfrac{1}{2}\left(\xi \frac{u_o}{V}\right)^2\right]^{\frac{1}{2}} \qquad (49)$$

used in the various methods was replaced by

$$\text{(a)} \quad \frac{V_{*wc}}{V_{*c}} = \left[1 + \tfrac{1}{2}\left(\xi \frac{u_o}{V}\right)^2 x\left(\frac{V}{u_o}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}, \quad \frac{u_o}{V} > 1 \qquad (50)$$

$$\text{(b)} \quad \frac{V_{*wc}}{V_{*c}} = \left[1 + \tfrac{1}{2}\left(\xi \frac{u_o}{V_o}\right)^2 x \frac{V}{u_o}\right]^{\frac{1}{2}}, \quad \frac{u_o}{V} > 1 \qquad (51)$$

$$\text{(c)} \quad \frac{V_{*wc}}{V_{*c}} = \left[1 + \tfrac{1}{2}\left(\xi \frac{u_o}{V_o}\right)^2 \frac{1}{\ln\left(\frac{u_o}{V}\right)}\right]^{\frac{1}{2}}, \quad \frac{u_o}{V} > 1 \qquad (52)$$

(d) Replacing u_o by $u_o x0.5m$ in equation (49) for $\frac{u_o}{V} > 1$ (53)

(e) Replacing u_o by $u_o x0.75$ in equation (49) for $\frac{u_o}{V} > 1$ (54)

(f) Replacing u_o by $u_o \left(\frac{V}{u_o}\right)^{\frac{1}{2}}$ in equation (49) for $\frac{u_o}{V} > 1$ (55)

The results of using the various expressions for shear velocity are shown in Figures 12 to 17. The behaviour of the various expressions depends in part into which transport equation it is inserted and on which data it is tested. The conclusions, therefore, must be tentative and their ad hoc nature acknowledged. With the Ackers and White-Swart equation the expression

$$\frac{V_{*wc}}{V_{*c}} = \left[1 + \tfrac{1}{2}\left(\xi \frac{u_o}{V}\right)^2 \frac{V}{u_o}\right]^{\frac{1}{2}}$$

provided the best general agreement with observations and provided acceptable predictions for $u_o/V < 3$. Transport rates for $u_o/V > 3$ were under predicted. The Ackers and White-Swart combined with the Rance equation provided the best agreement between predicted and observed results, see Figure 18. The uncertainties in the data mean that it is difficult to assess the confidence limits associated with the various equations. The results indicate, however, that even by the standards of sediment transport the predictions of the thoeries are not good. For the Ackers and White-Swart and Rance equations 46% of the predictions were within a factor of 4 of the observed value and 69% were within a factor 10.

8 CONCLUSIONS AND RECOMMENDATIONS

A number of theories for sediment transport under waves and currents have been compared with field and laboratory observations. The theories were all extensions of formulae for sediment transport under uni-directional flow in which the expression (1) for the bed shear stress under waves and currents had been inserted. The predictions of the various theories were, in general, unreliable except when the effect of currents dominated that of the waves. It was found that the predictions could be improved if the equation for the bed shear stress (1) was replaced by equation (4). Of the various theories considered those based on Ackers and White sediment transport theory seemed to show marginally better predictions.

Since the over-prediction of the various methods appeared to increase as the effect of the waves increased relative to that of the currents a number of modifications to the bed shear stress were investigated which reduced the shear as the waves increased (see eqs 45 to 50). These modifications, however, lead only to minor improvements in the predictions.

A weighted combination of the Ackers and White-Swart equations and the Rance equation for sediment transport under waves was tried in which the weighting function ensured that for no waves the equations reduced to the Ackers and White equations and for no currents the equations were identical to the Rance equations. These equations gave the best performance on the Bijker and Boscombe pier data. Their performance on the Inman and Bowen data was less satisfactory, see Table 3.

It is expected that were a more reliable equation for sediment transport under waves and currents available as the asymptote in the waves only case then better predictions would result. Until such equations become available it is recommended that a weighted combination of the Ackers and White-Swart and Rance equations are used. Consideration, however, should be given to how the range of parameters in any application compare with those of the data on which the Rance equation was developed.

A major weakness in the present theories for sediment transport under waves and currents appears to be the definition of the bed shear stress. It is, therefore, recommended that further attention is given to equations that describe the bed shear stress under waves and currents.

To remove the uncertainty about the proper form of asymptote in the waves only case further work is required on a theory to describe sediment transport under waves alone.

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10 NOTATION

A		Value of sediment mobility at initiation of motion (Ackers and White)
a		Constant in approximation of τ_{wc}/τ_c
a_o	m	Amplitude of orbital excursion at the bed
b		Constant in approximation to τ_{wc}/τ_c
C	$m^{\frac{1}{2}}s^{-1}$	Chezy coefficient
c		Exponent in approximation to τ_{wc}/τ_c
$C_{D_{90}}$	$m^{\frac{1}{2}}s^{-1}$	Chezy coefficient based on D_{90} sediment size
D_{50}, D_{90}	m	Sediment diameter of which 50% (90%) is finer
d	m	depth of water
F_{gr}		Sediment mobility (Ackers and White)
G_{gr}^{wc}		Sediment mobility under waves and currents (Ackers and White)
f_w		Jonsson wave friction factor
G	m/s	Sediment transport rate
G_{gr}		Dimensionless sediment transport rate (Ackers and White)
g	m/s^2	Acceleration due to gravity
n		Transition exponent in sediment transport function (Ackers and White)
P, P_{fg}, P_{cg}	kg/s^3	Power per unit area available to move sediment, (fine grain, coarse grain)
P_B		Constant = 0.45
Q_s	m^2/s	Sediment transport rate
r	m	Bed roughness
S	m^2s^{-1}	Bed load, sediment volume transport rate
s		Specific gravity of sediment
T	s	Wave period
t	s	Time
u_0	m/s	Wave orbital velocity at bed
V	m/s	Current velocity
V_{TOT}	m/s	Instantaneous water velocity, Willis theory
V_*	m/s	Shear velocity due to current
V_{*wc}	m/s	Shear velocity due to waves and current
W_c		Empirical constant in Ackers and White - Willis

X		Sediment concentration by weight
α		Angle between current wave directions
β		V_{*wc}/V_{*c}
ϕ		Angle between wave direction and normal to the current
κ		Van Karman's constant
μ		$(C/C_{D_{90}})^{3/2}$, Frijlink-Bijker theory
ξ		Parameter depending on the bed roughness and water depth
ρ	kg/m^3	Density of water
ρ_s	kg/m^3	Density of sediment
τ_c	$kg/m/s^2$	Shear stress under currents
τ_{wc}	$kg/m/s^2$	Shear stress under waves and currents
ψ		Shields function (Sleath)
ψ_c		Critical value of Shield's function
ω	s^{-1}	Wave frequency

Tables

TABLE 1 : COMPARISON OF EXPRESSIONS FOR τ_{wc}/τ_c

$\xi \frac{u_o}{V}$	$\frac{\tau_{wc}}{\tau_c}$ Exact integral $\theta = 90°$	$\frac{\tau_{wc}}{\tau_c} = 1 + \frac{1}{2} (\xi \frac{u_o}{V})^2$ (Swart) $\theta = 90°$	$\frac{\tau_{wc}}{\tau_c} = 0.75 + 0.45 (\xi \frac{u_o}{V})^{1.13}$ (Graaf and Overeem) $0 < \theta < 20°$
0	1.0	1.0	0.75
1	1.5	1.05	1.20
2	2.65	3.00	1.73
10	12.75	51.00	6.82
100	127	5001	82.6

TABLE 2 : SEDIMENT TRANSPORT UNDER WAVES

	Mean Discrepancy Ratios		
	Sleath 1978/1982	Sleath 1978	Rance 1971
Data			
Shibayama and Horikawa	18.9	17.3	2.83
Vincent : fine sand	0.48	0.14	0.02
Vincent : medium sand	3.55	1.81	0.0003
Vincent : pumice	2.70	2.70	-

TABLE 3 : SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS

MEAN DISCREPANCY RATIOS

SEDIMENT TRANSPORT THEORY	**BIJKER**	**BOSCOMBE PIER (MODIFIED)**	**INMAN & BOWER (MODIFIED)**
Bijker	10.7	62.3	963
Ackers, White, Willis	9.08	1421.0	5294
Ackers, White, Graaf	4.23	121.5	1544
Ackers, White, Swart	3.3	32.4	3.86
Engelund, Hansen, Swart	27.3	64.9	3423
WITH BIJKER SHEAR STRESS INTEGRATION			
Bijker	9.60	42.8	2.62
Ackers, White, Willis	1.82	52.5	-
Ackers, White, Graaf	2.58	15.0	1.09
Ackers, White, Swart	3.08	17.7	-
Engelund, Hansen, Swart	18.59	17.1	80.4
Ackers, White, Swart (Modified)	3.4	19.2	-
Ackers, White, Swart (with Bijker integration & modified)	3.08	16.5	-
Ackers, White, Swart and Rance	1.96	8.34	52.3

Figures

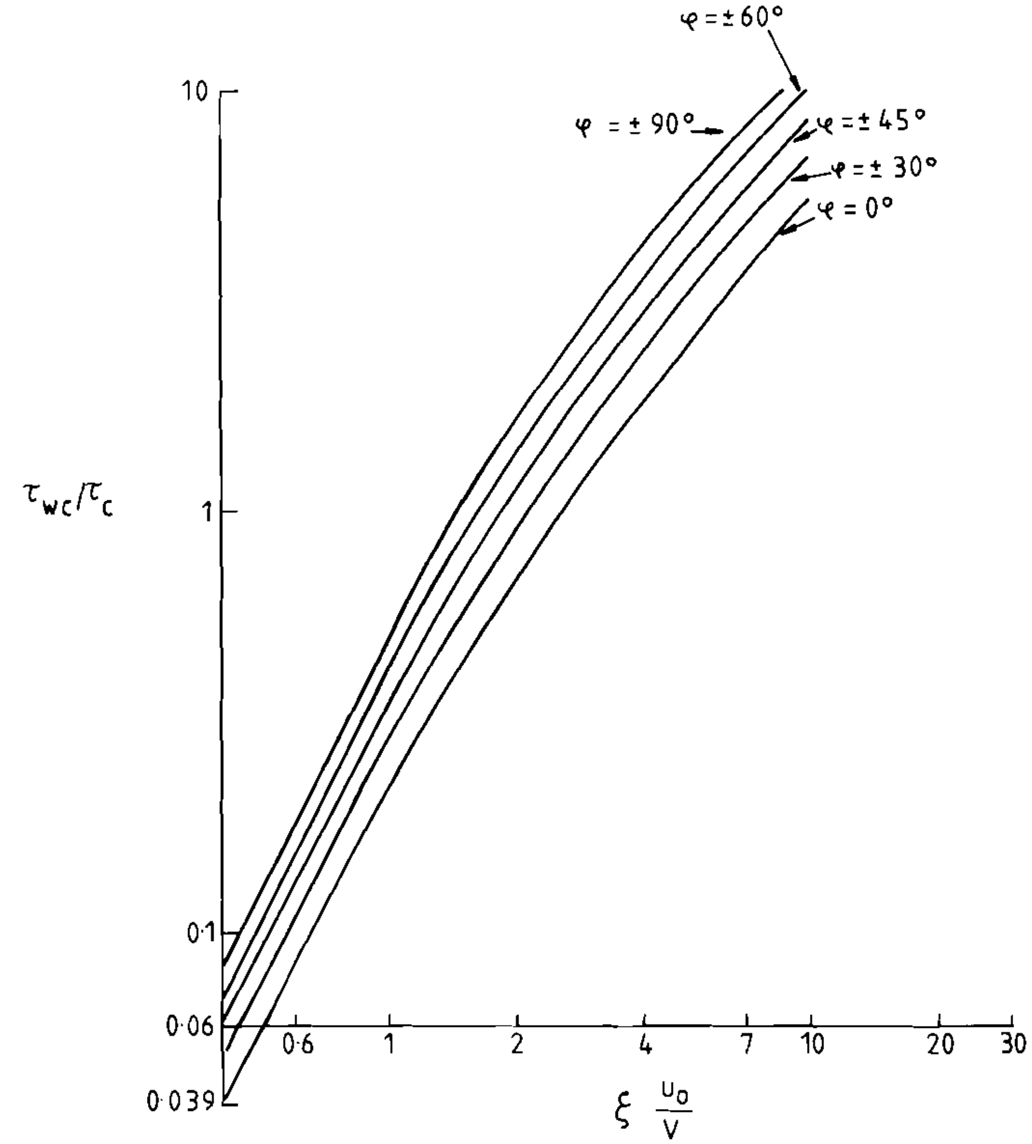

Fig 1 Computed values of τ_{wc} / τ_c (from Bijker 1967)

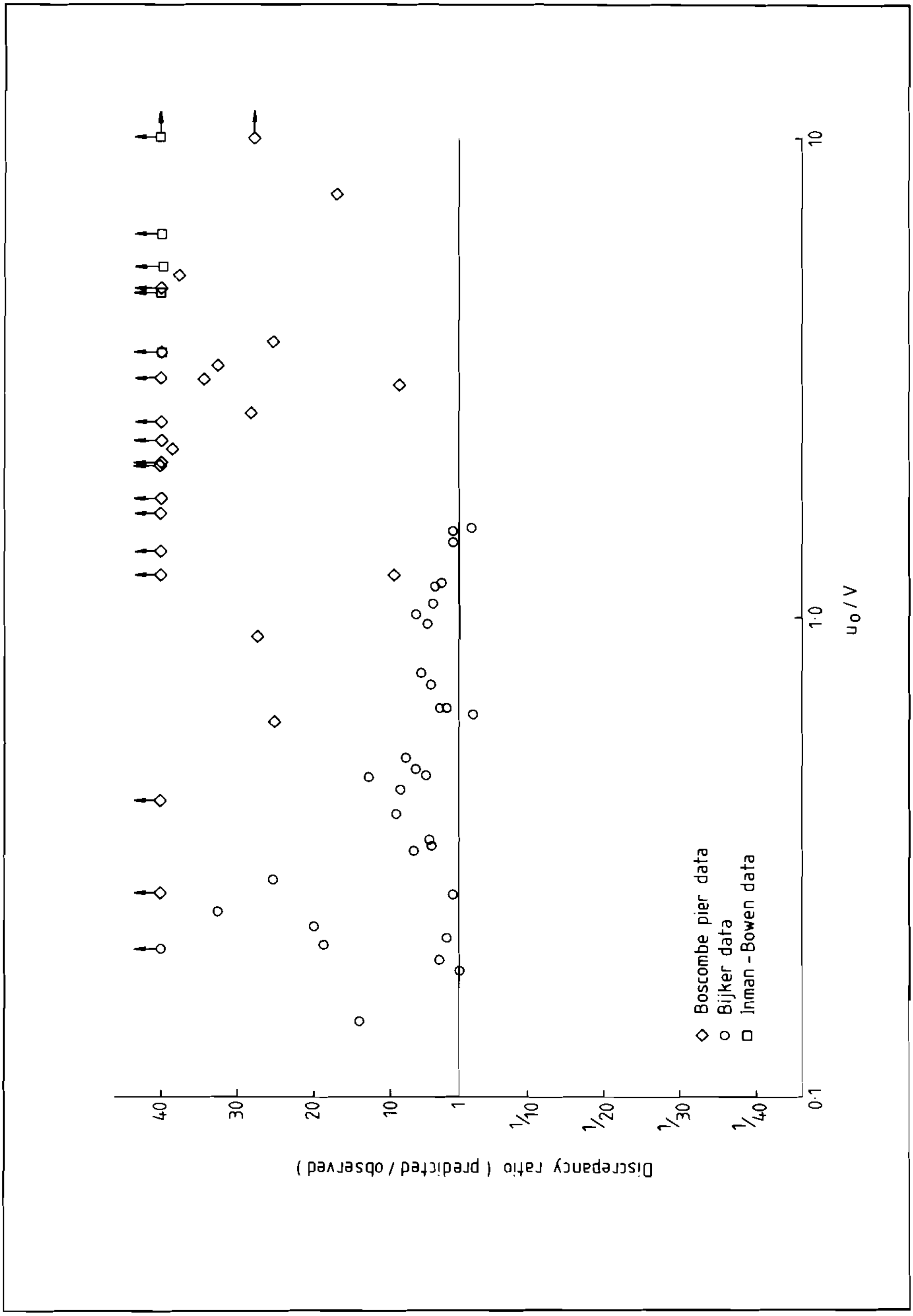

Fig. 2 Discrepancy ratios, Frijlink - Bijker

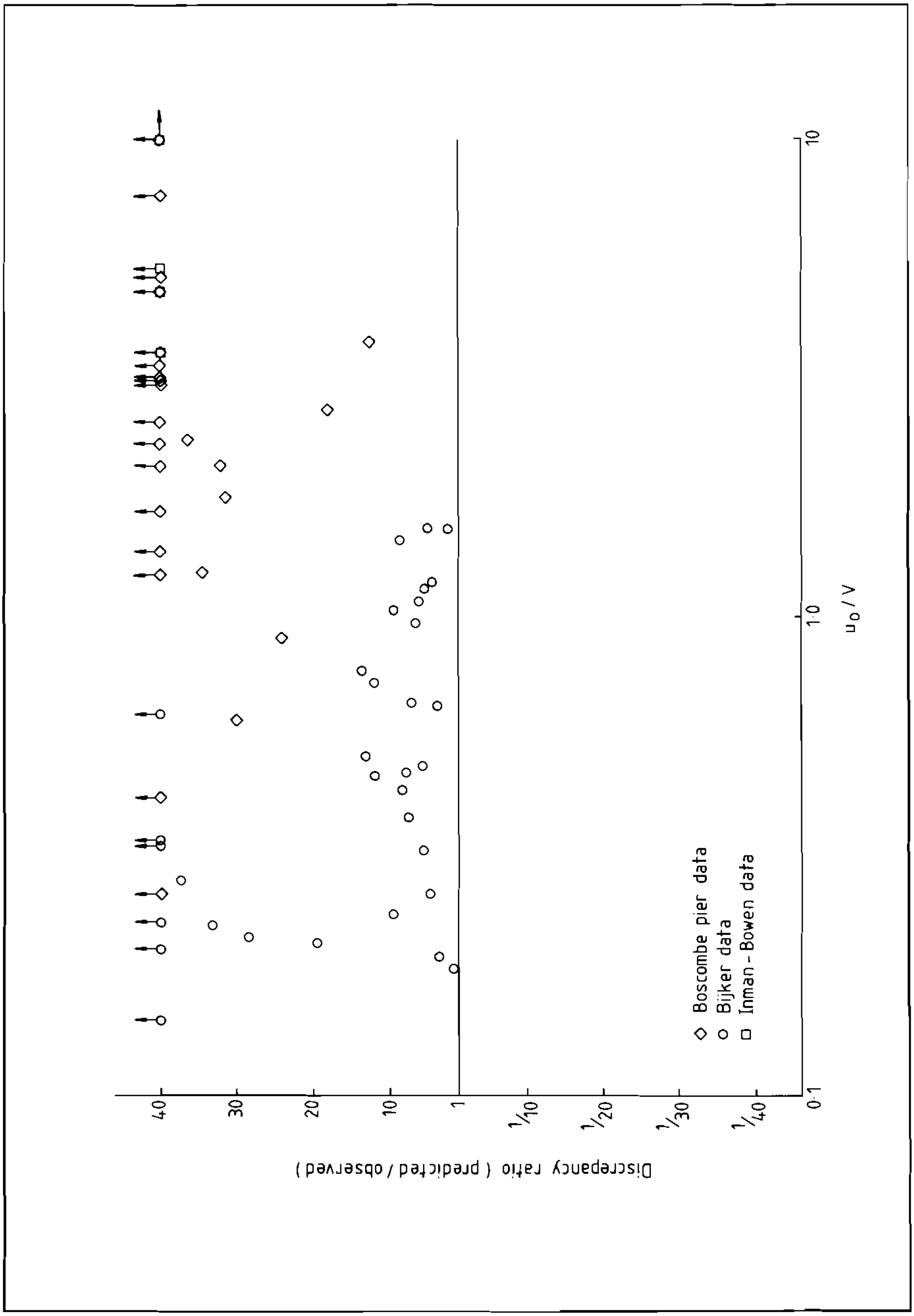

Fig. 3 Discrepancy ratios, Engelund and Hansen - Swart

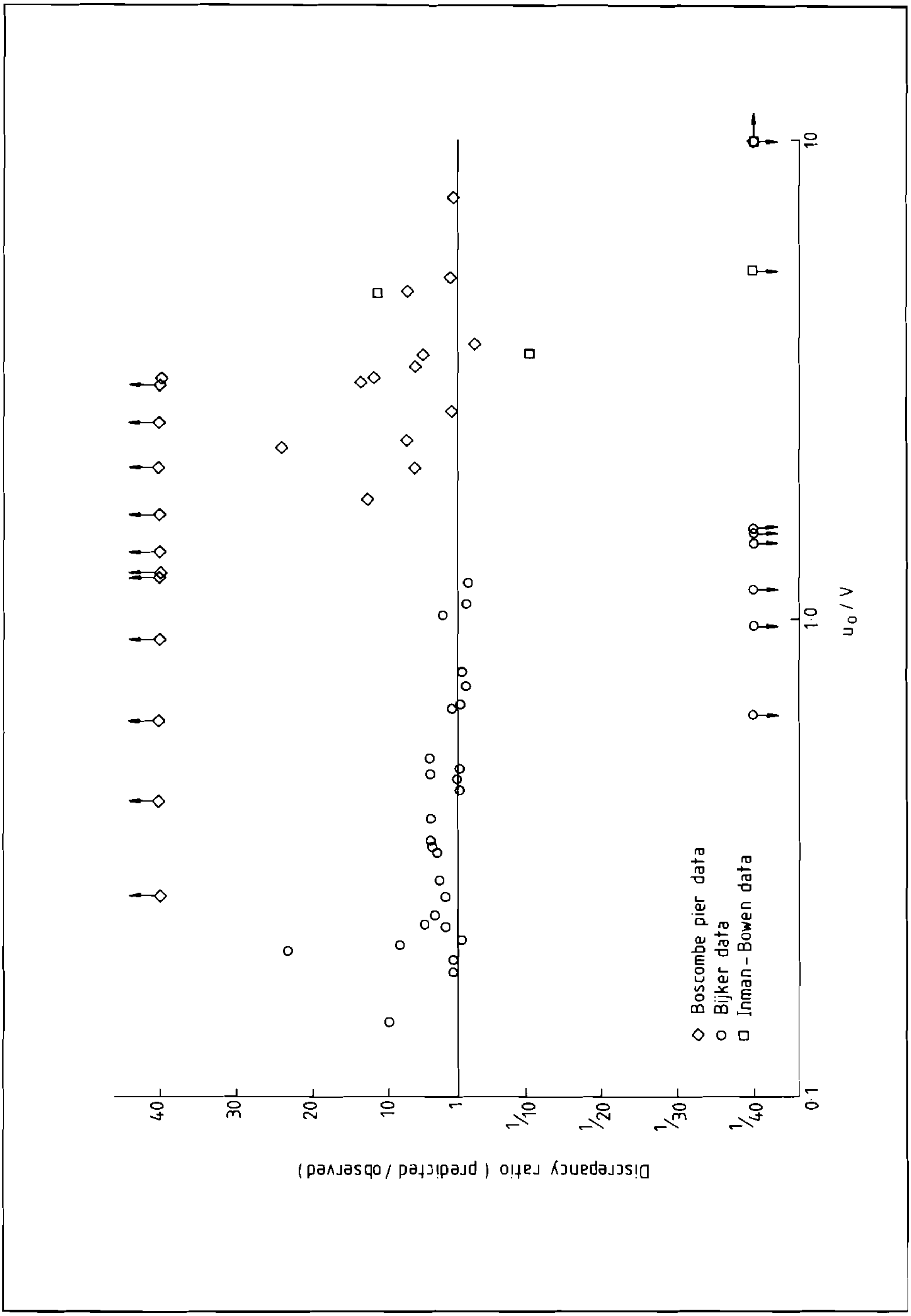

Fig. 4 Discrepancy ratios, Ackers and White - Swart

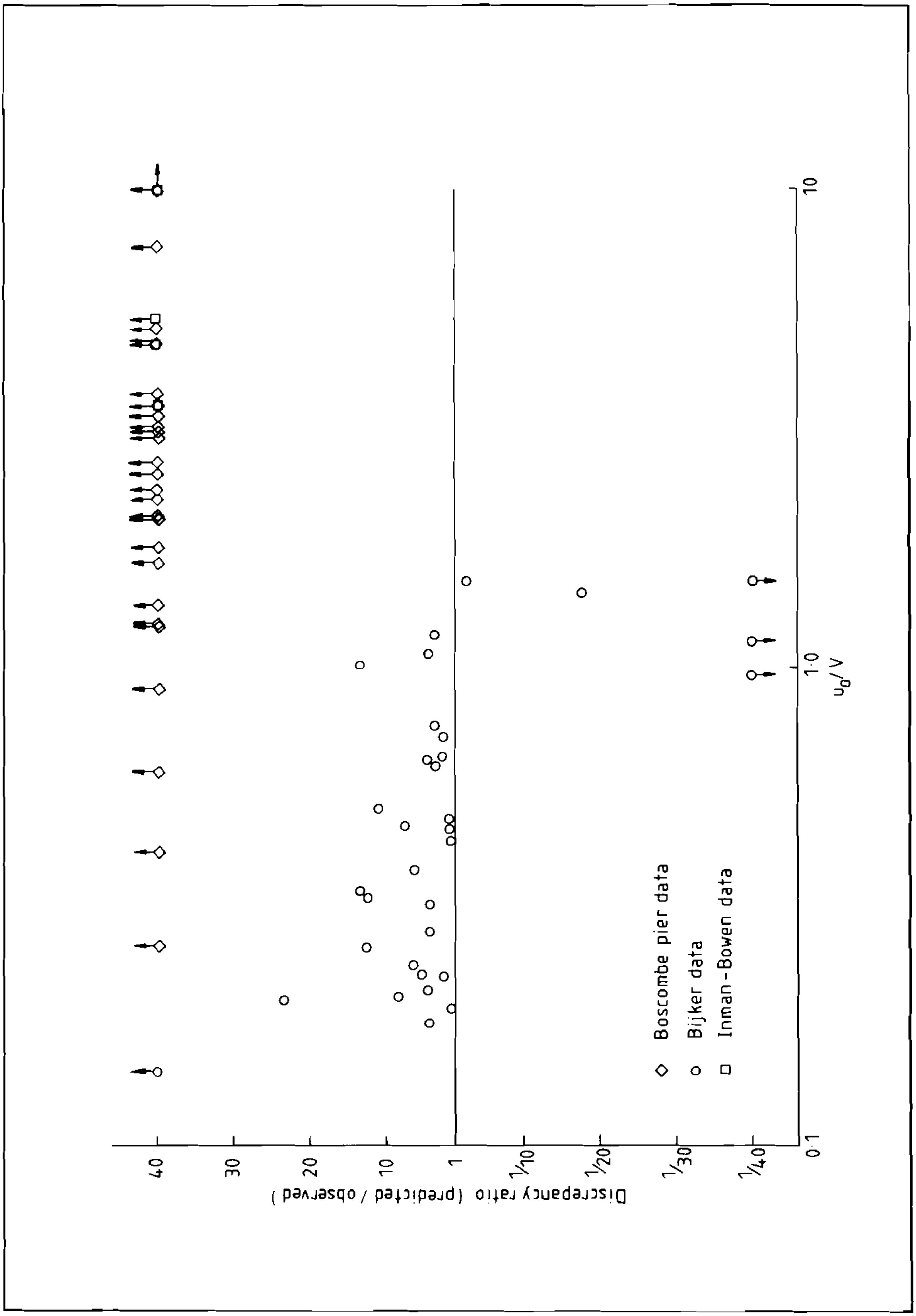

Fig 5 Discrepancy ratios, Ackers and White - Willis

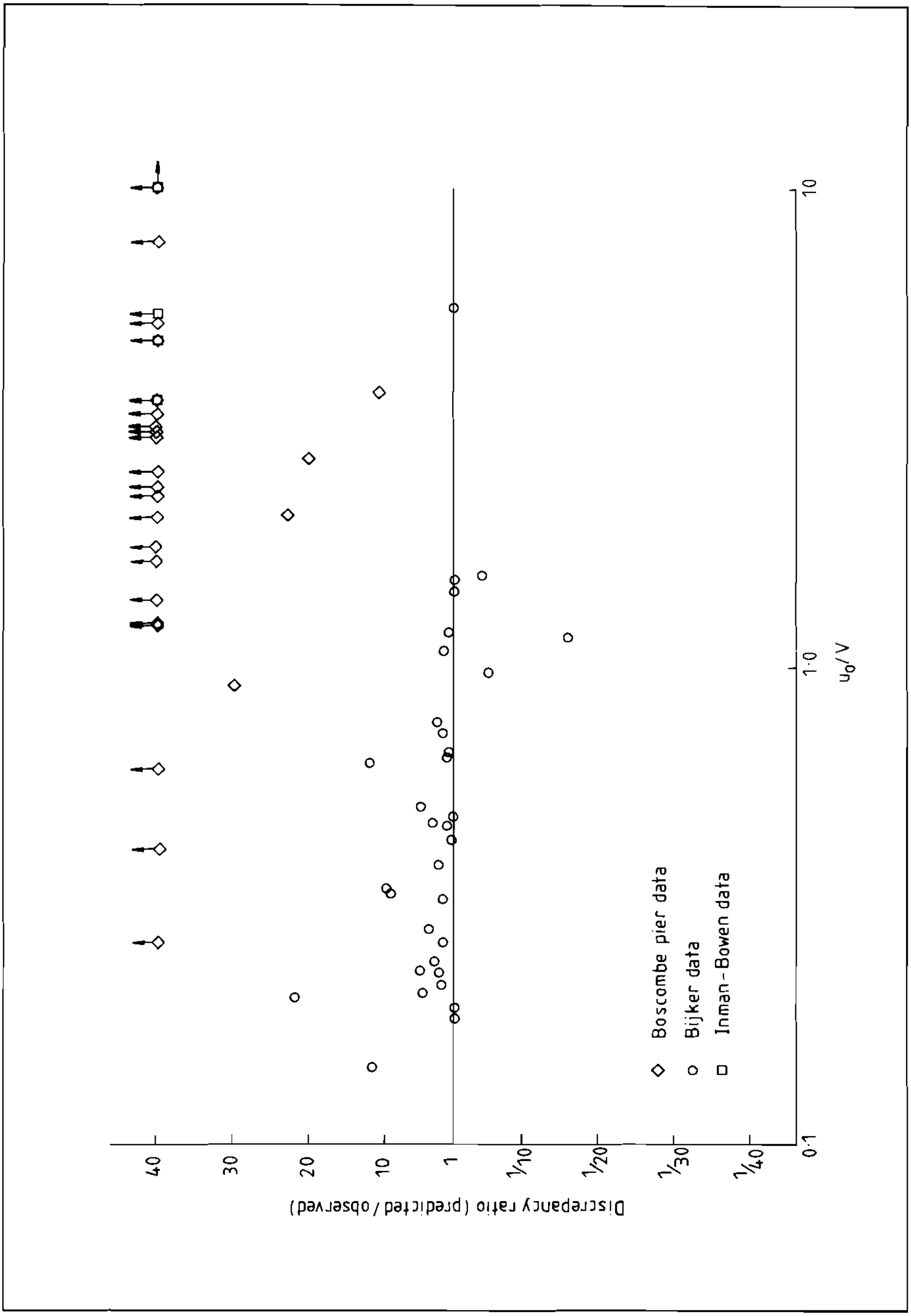

Fig 6 Discrepancy ratios, Ackers and White - Graaff and Overeem

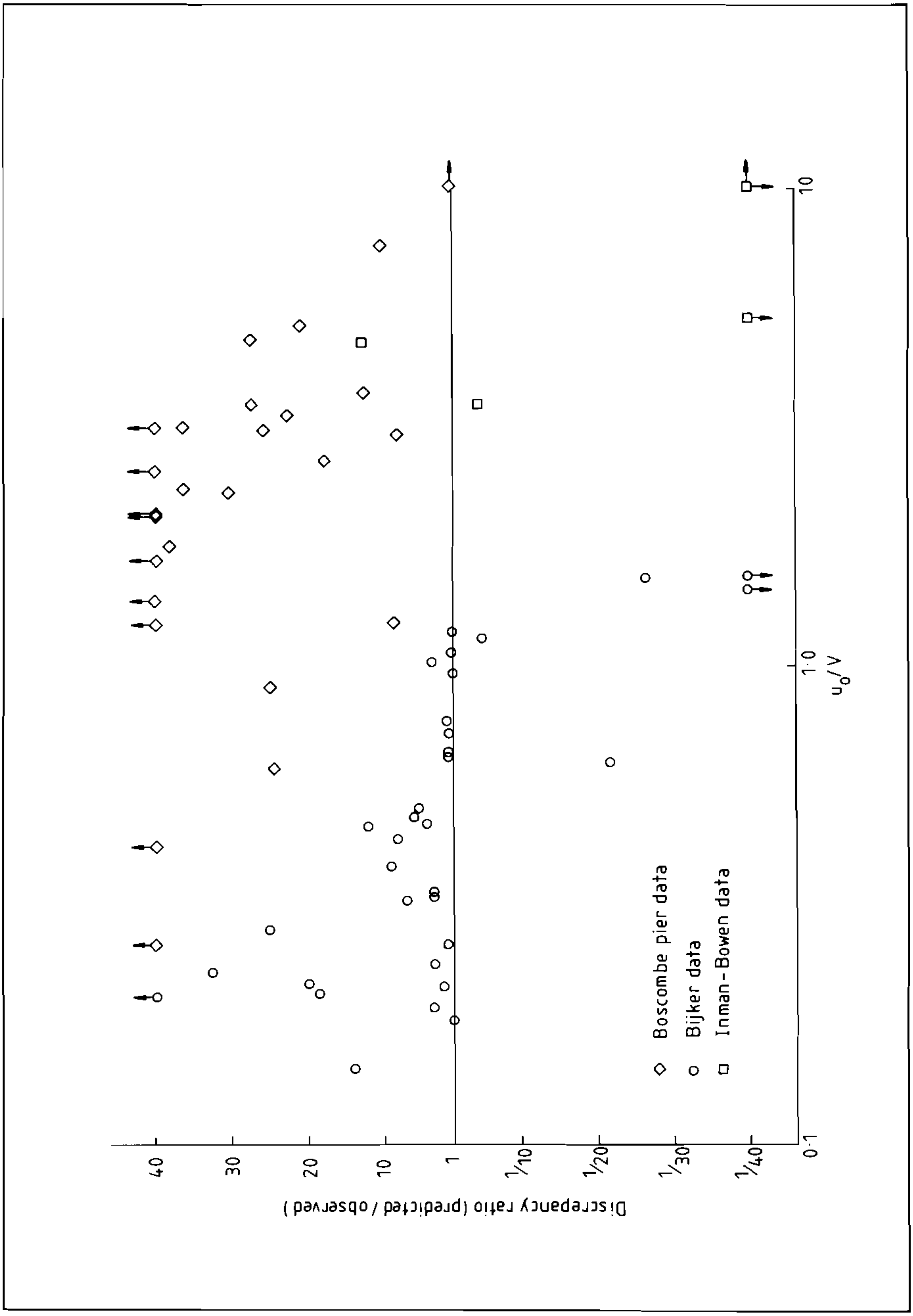

Fig 7 Discrepancy ratios, Frijlink - Bijker with Bijker integration

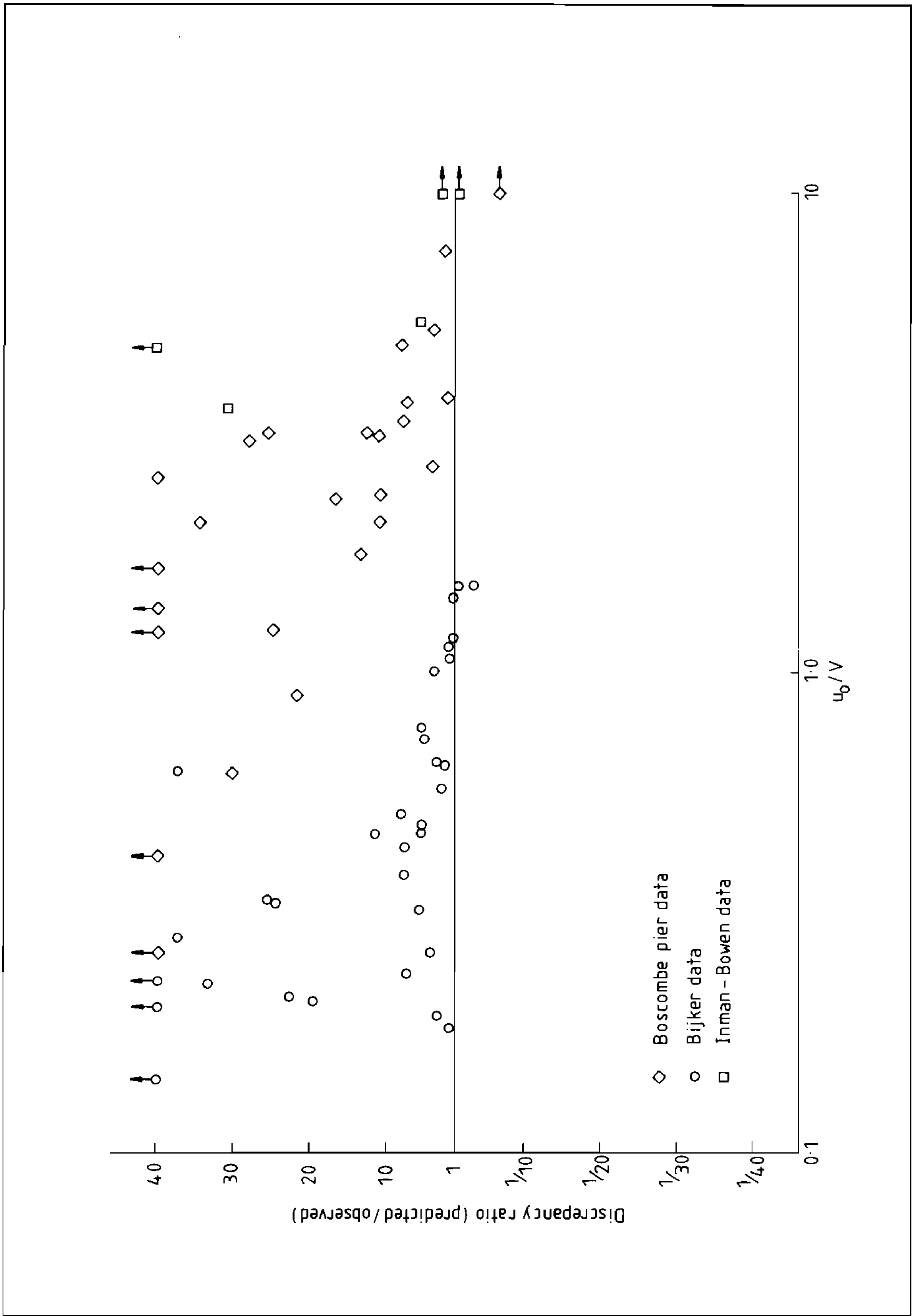

Fig 8 Discrepancy ratios, Engelund and Hansen - Swart with Bijker integration

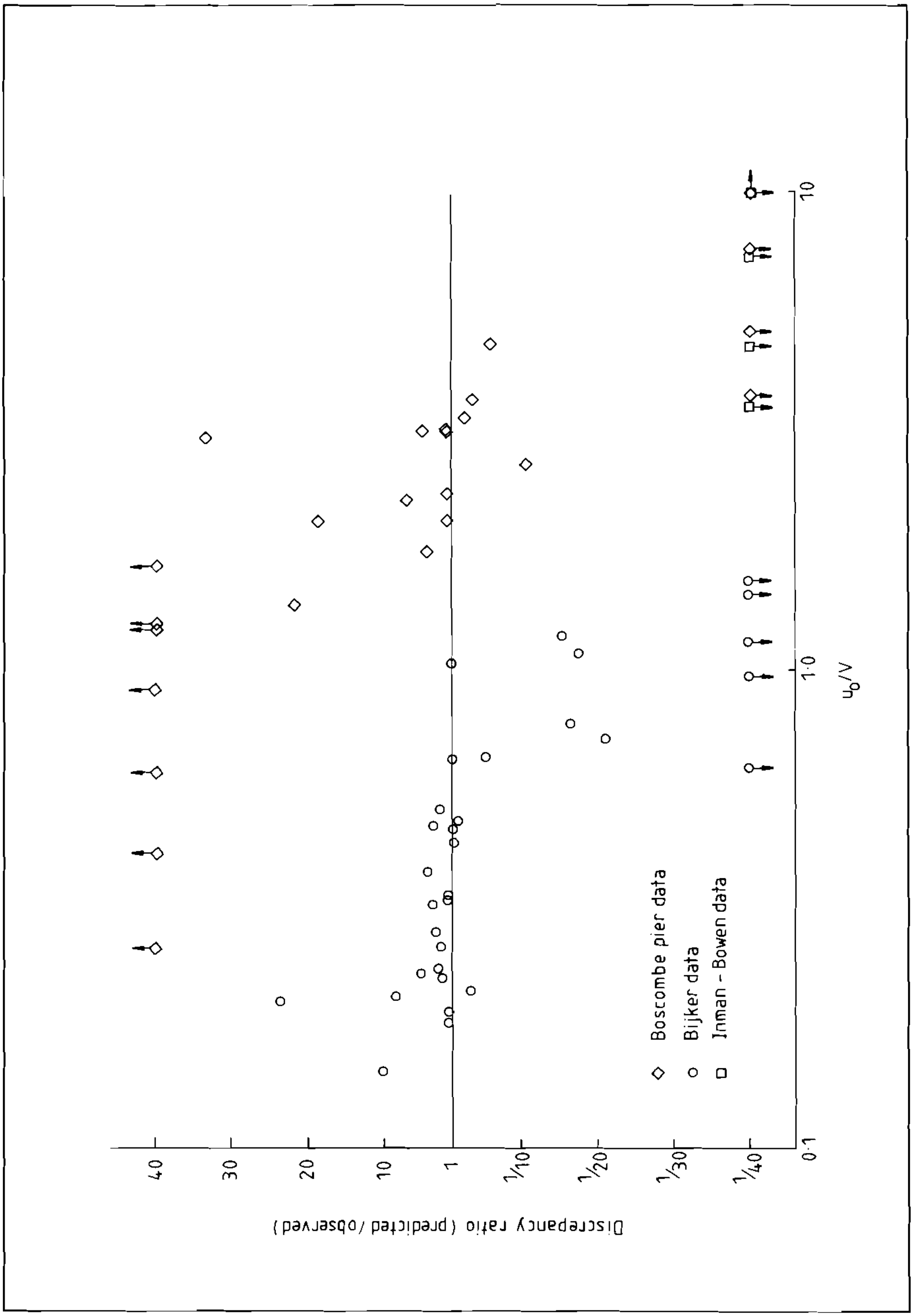

Fig 9 Discrepancy ratios, Ackers and White – Swart with Bijker integration

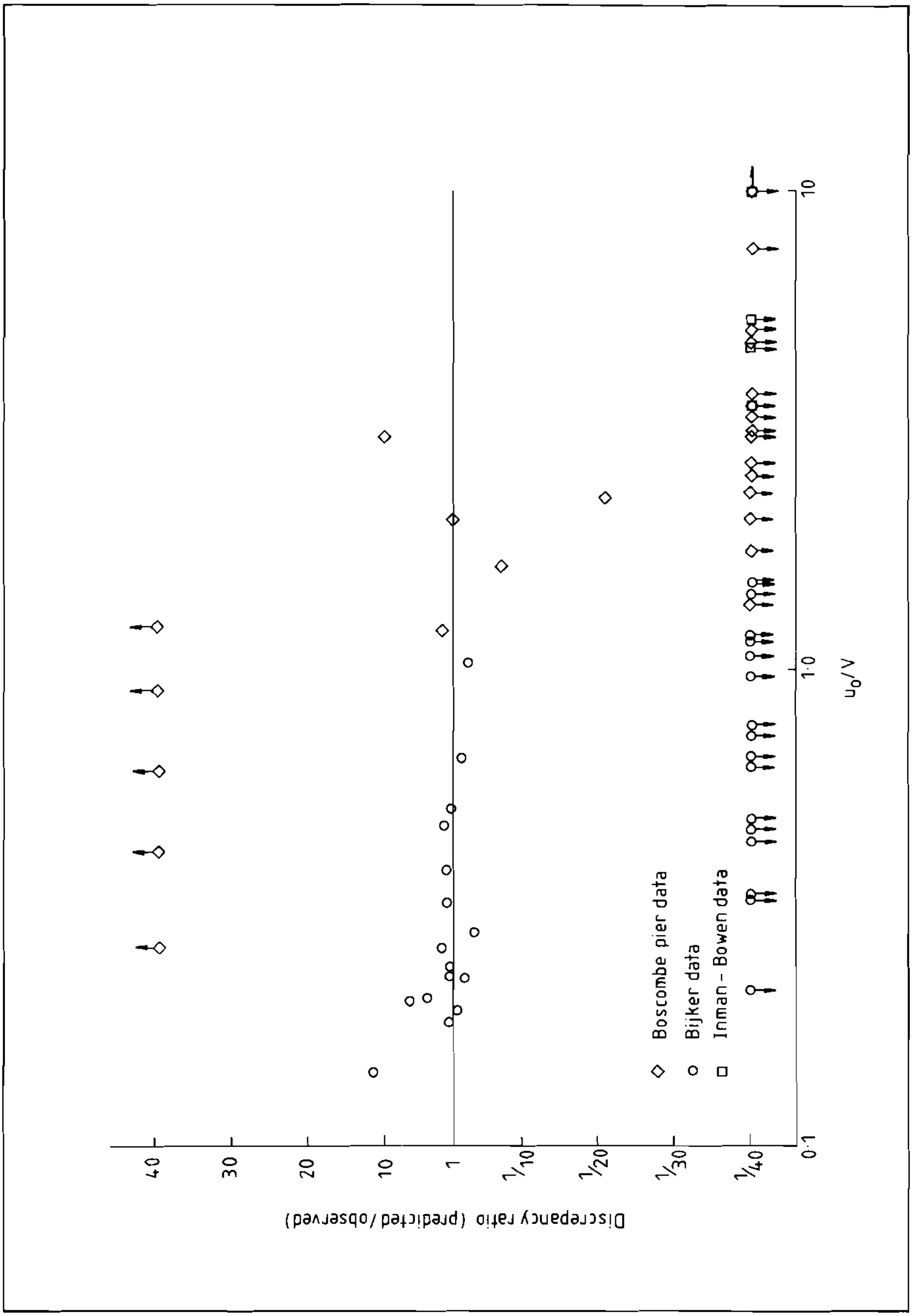

Fig 10 Discrepancy ratios, Ackers and White - Willis with Bijker integration

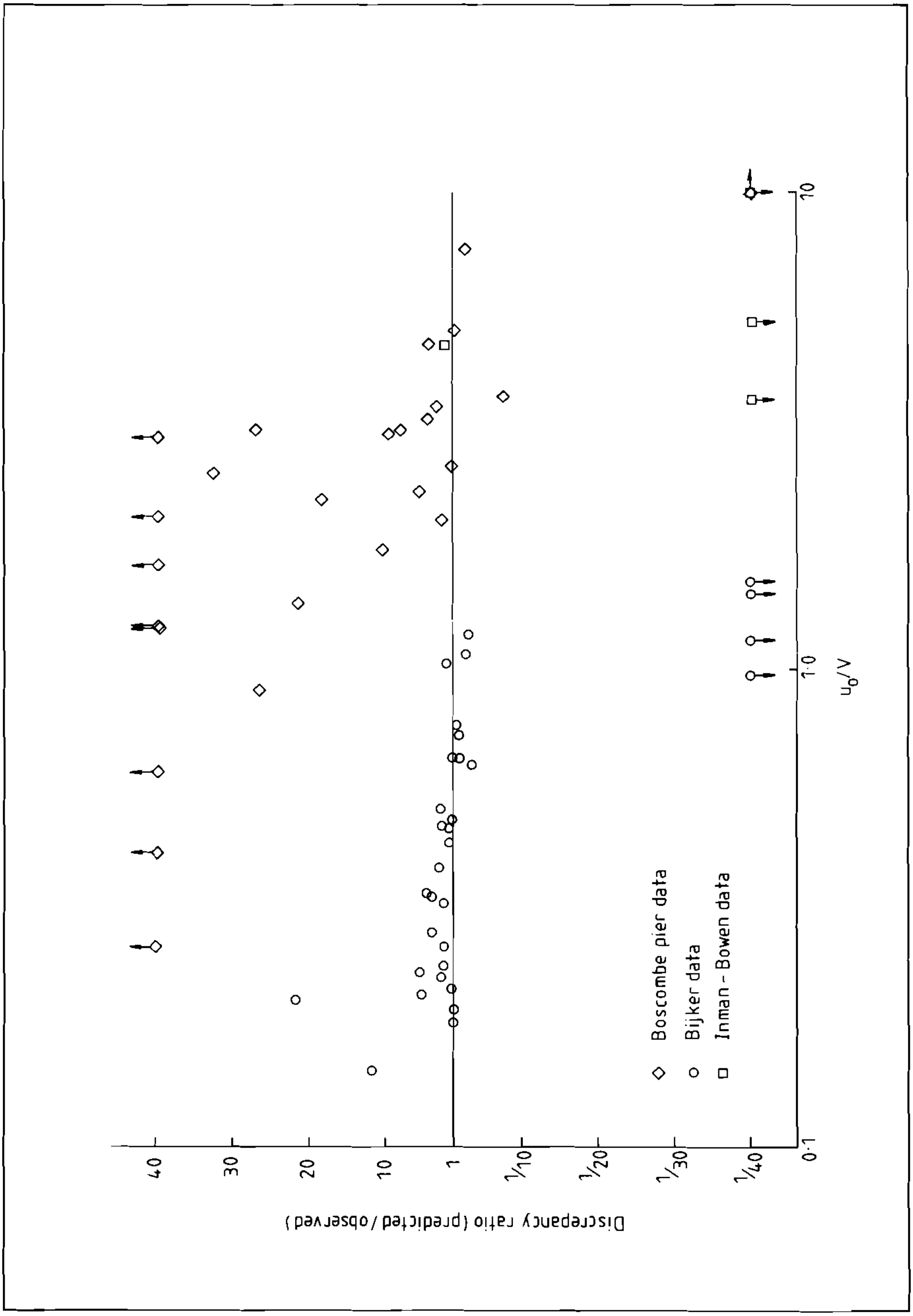

Fig 11 Discrepancy ratios, Ackers and White - Graaff and Overeem with Bijker integration

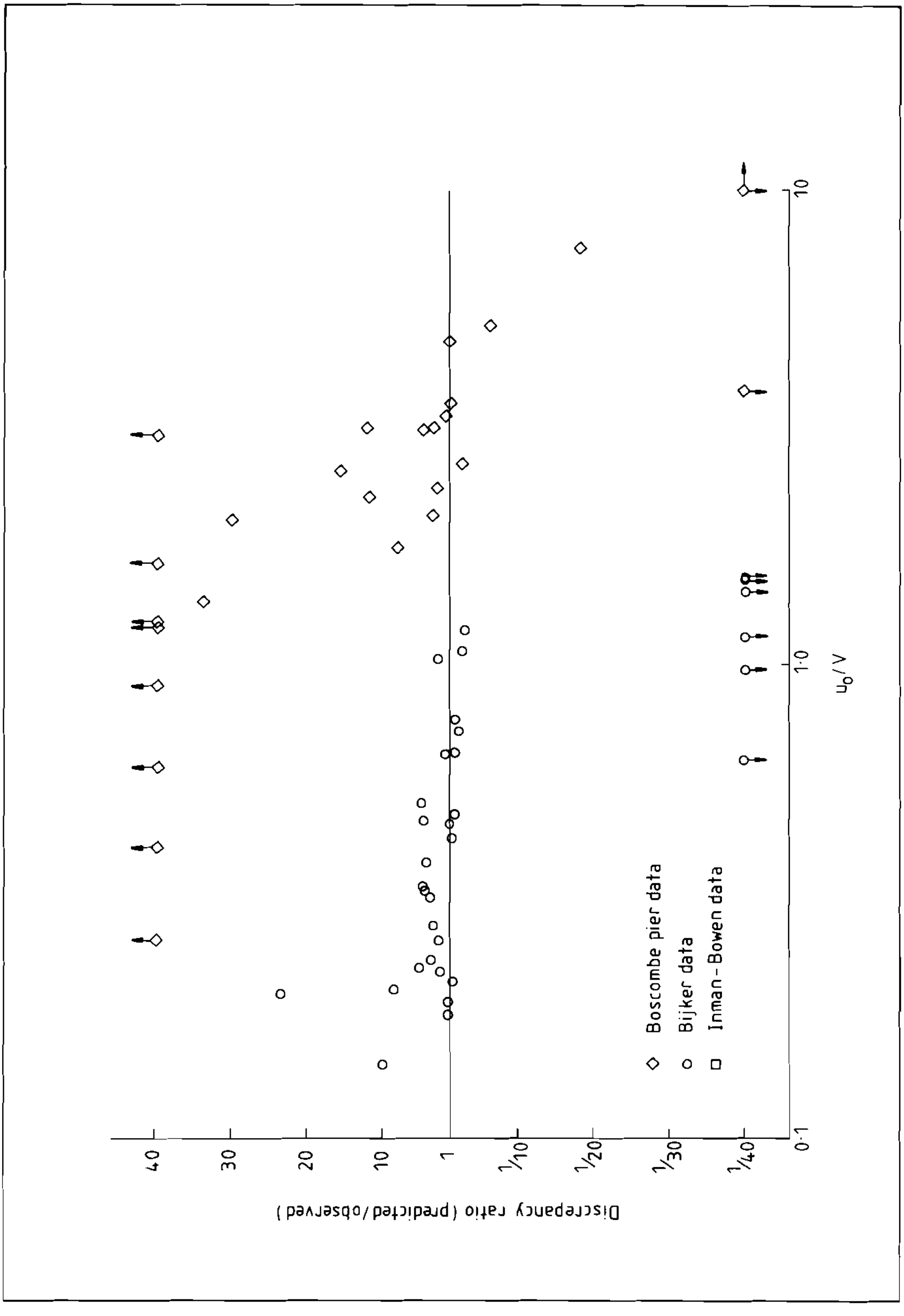

Fig 12 Discrepancy ratios, Ackers and White-Swart with Bijker integration and equation (50)

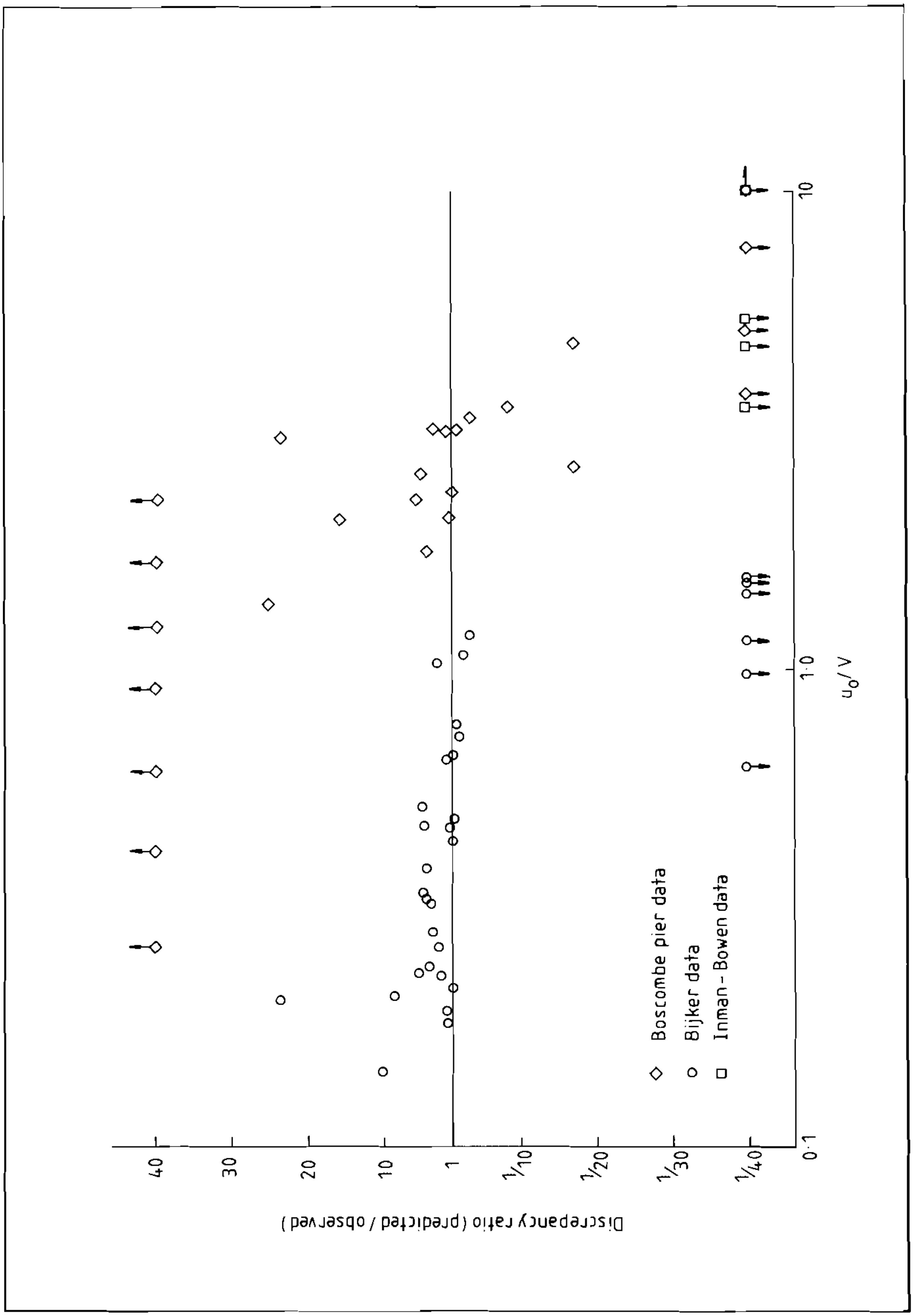

Fig 13 Discrepancy ratios, Ackers and White - Swart with Bijker integration and equation (51)

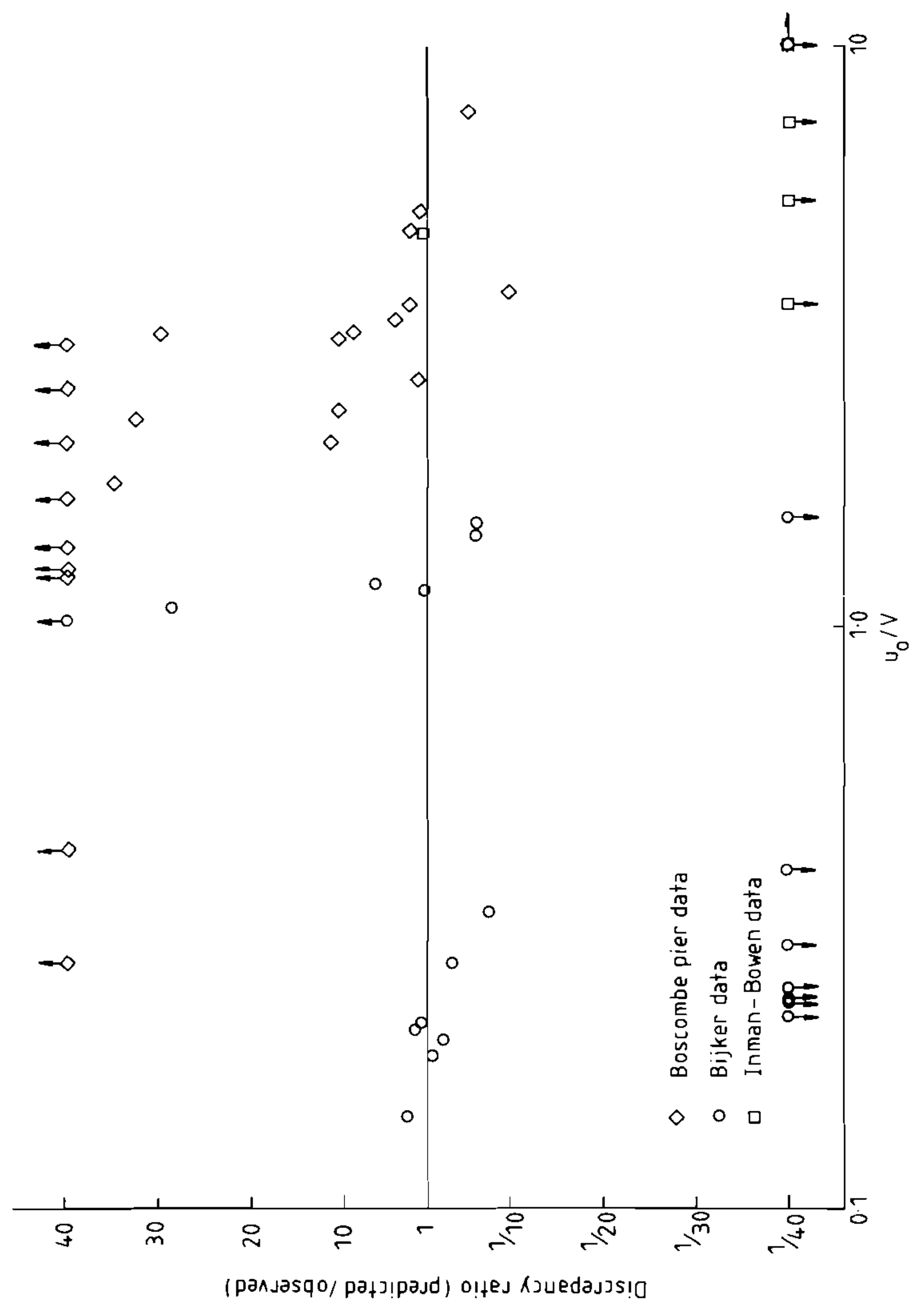

Fig 14 Discrepancy ratios, Ackers and White-Swart with Bijker integration and equation (52)

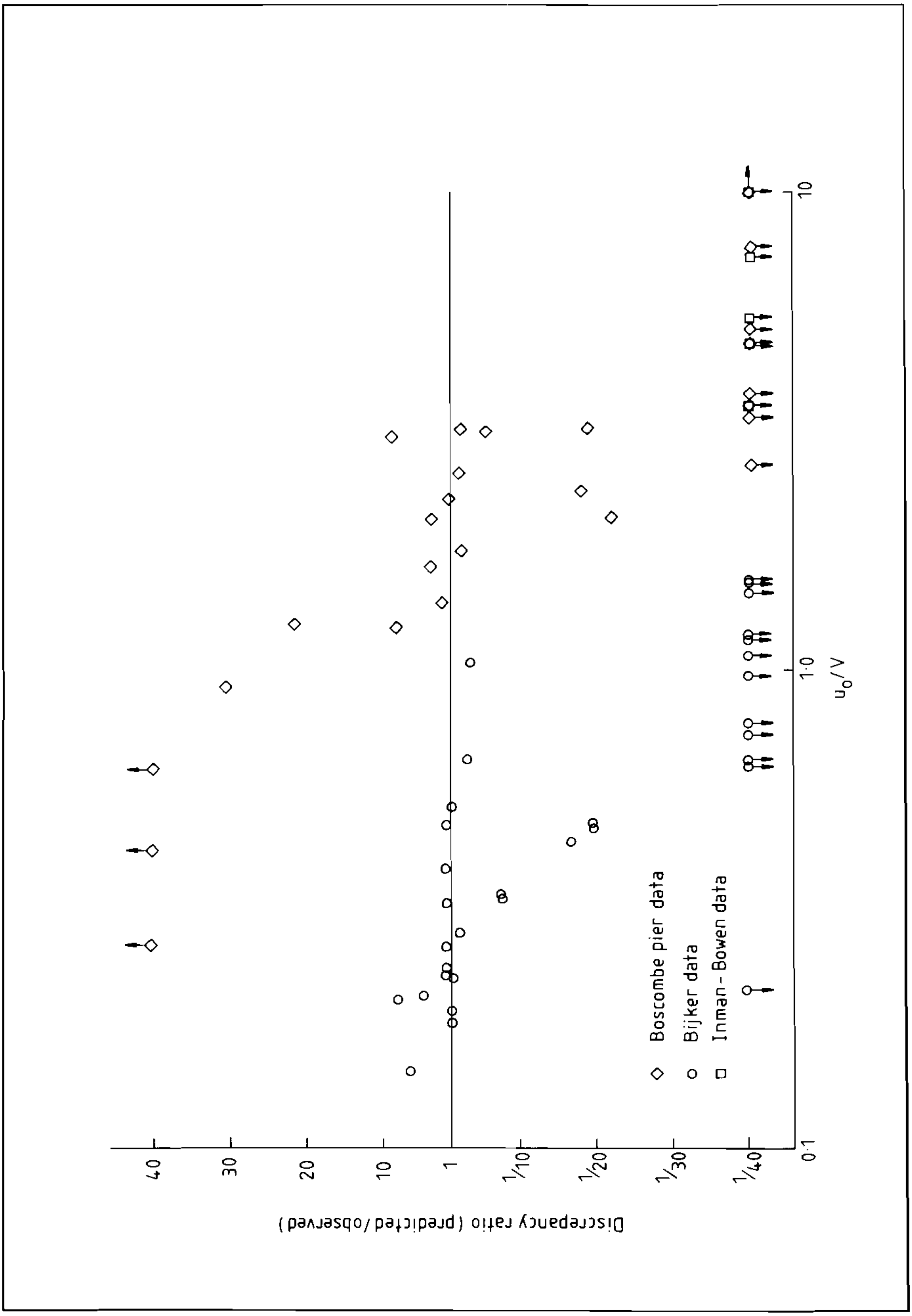

Fig 15 Discrepancy ratios, Ackers and White - Swart with Bijker integration and equation (53)

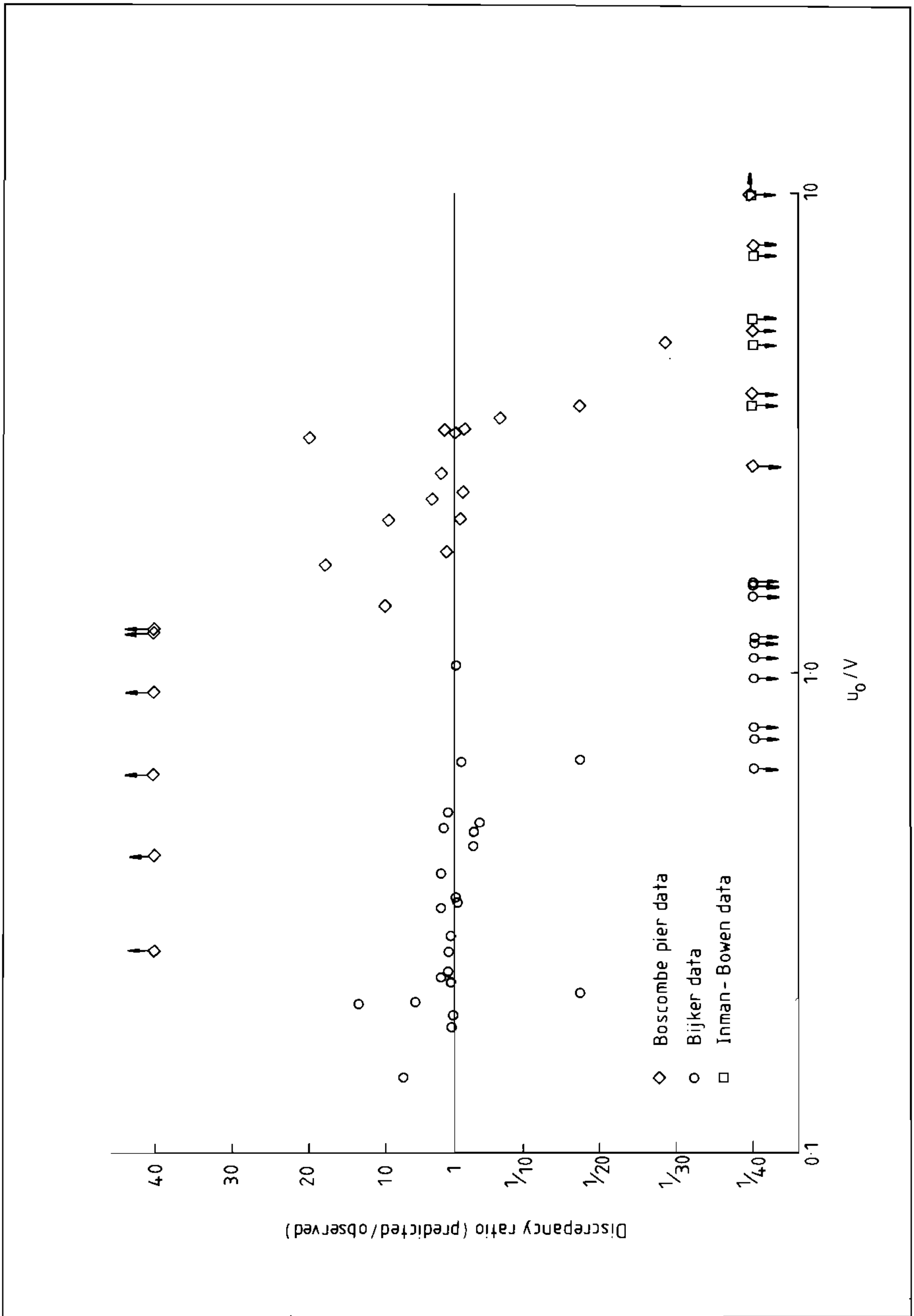

Fig 16 Discrepancy ratios, Ackers and White - Swart with Bijker integration and equation (54)

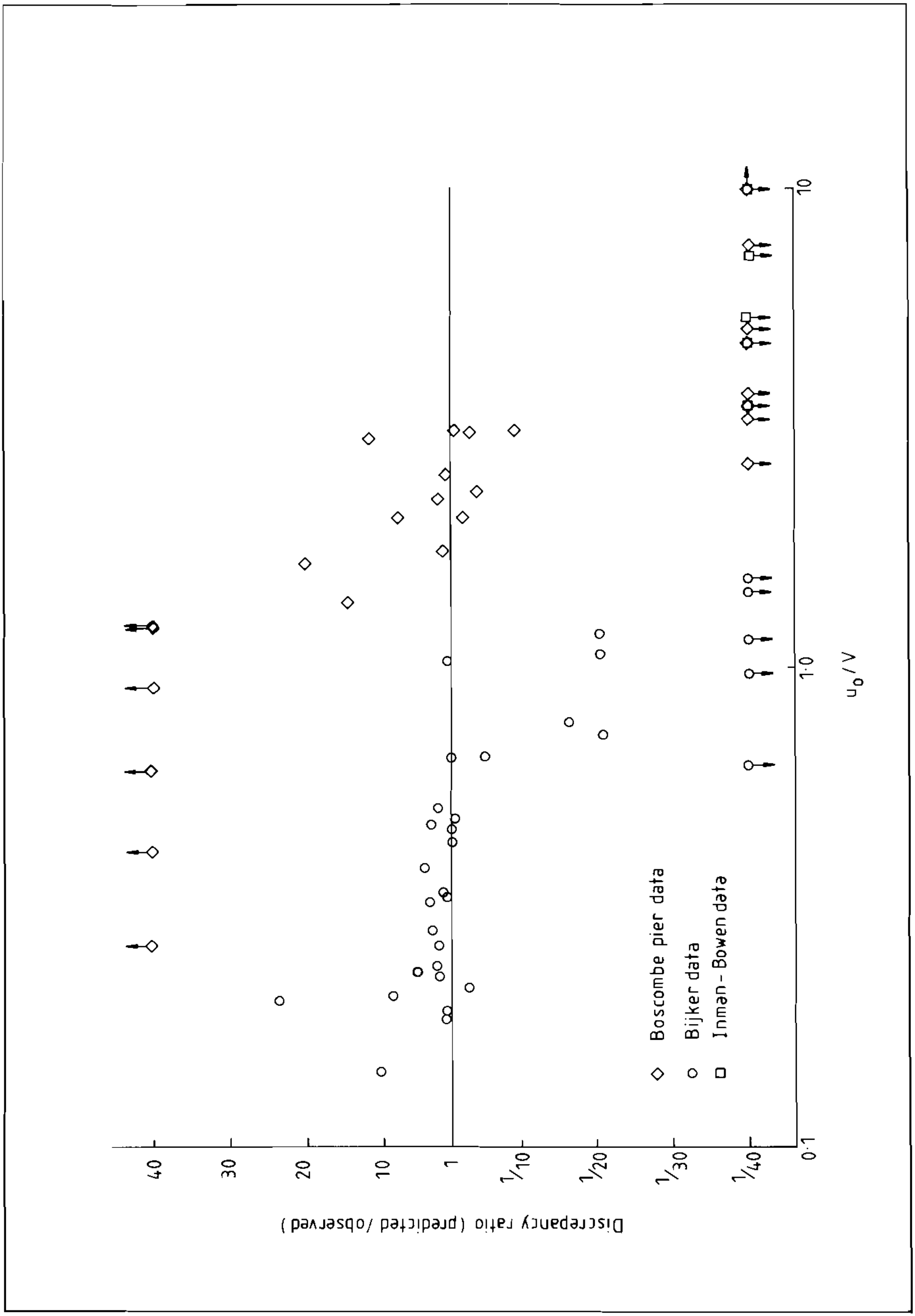

Fig 17 Discrepancy ratios, Ackers and White - Swart with Bijker integration and equation (55)

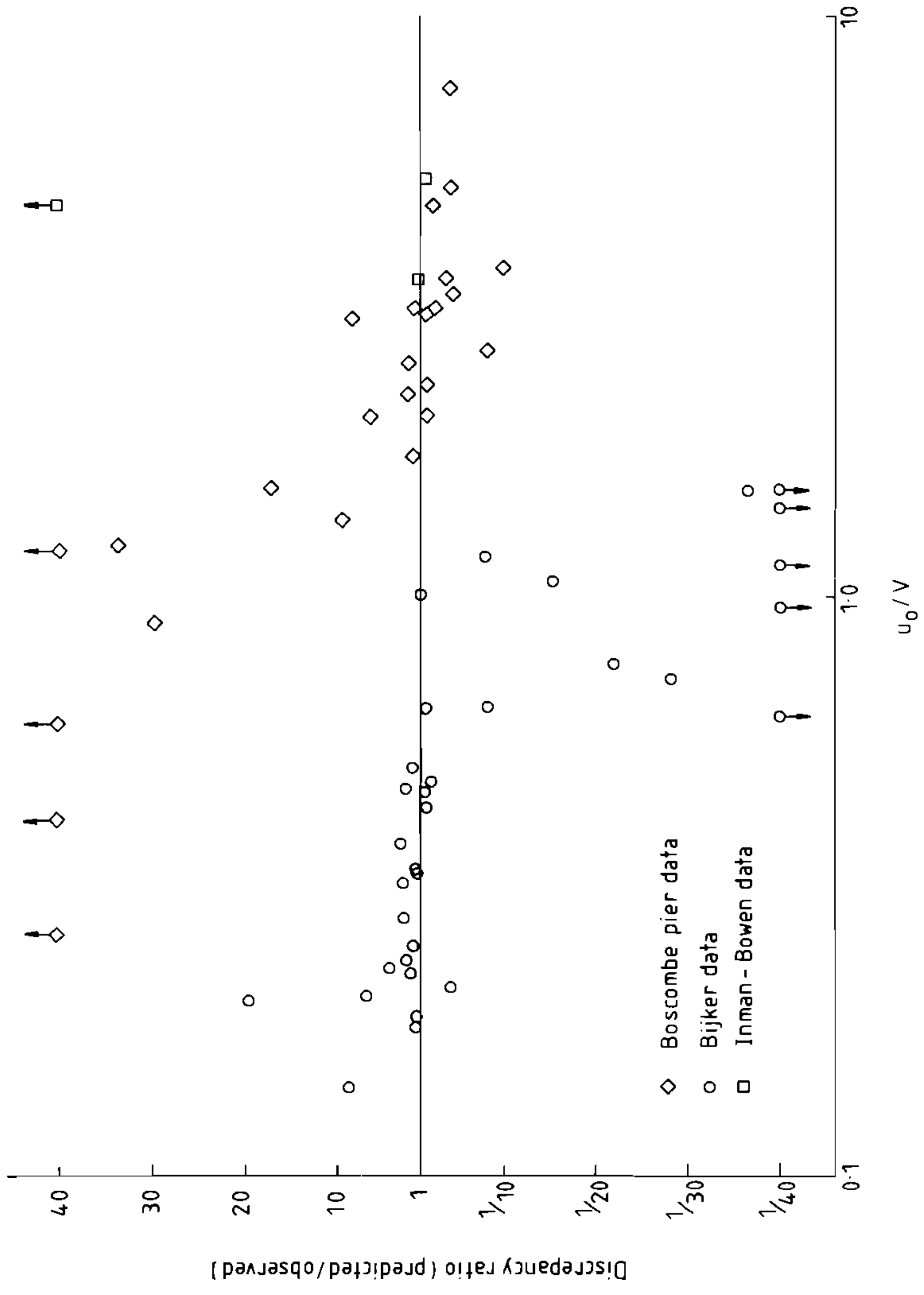

Fig 18   Discrepancy ratios, Ackers and White - Swart, Rance with Bijker integration