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NUMERICAL MODELLING OF SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS IN ESTUARIES

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Registered Office: Hydraulics Research Limited,
Wallingford, Oxfordshire OX10 8BA.
Telephone: 0491 35381. Telex: 848552

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ABSTRACT

The movement of sediments in estuaries frequently causes problems. Any engineering works undertaken in any estuary may affect the pattern of sediment erosion or deposition and lead to beneficial or deleterious affects at locations throughout the estuary, ocassionally at places remote from the site of the original engineering works. Thus the construction of docks, quays or jetties may lead to increased sediment deposition or alternatively erosion. The maintenance or enlargement of existing shipping channels may have an impact on sediment movement and any alterations in fluvial flow or tidal storage may affect patterns of sediment erosion or deposition. Before embarking upon such engineering works it is therefore important for their impact on an estuary to be assessed to enable rational decisions to be made about the advisability of carrying out proposed schemes and to assess which schemes will achieve the required aims. Because of the complexity of the physical system representing an estuary recourse has frequently to be made to numerical models to make such predictions. Numerical models have been successfully used in the past to predict the behaviour of both cohesive and non-cohesive sediments in estuaries but these models have ignored the effects of waves which in certain cases are significant. In this report the effects of waves in these situations are discussed and methods of including these in a numerical model are considered. Much is known separately about the behaviour of waves and currents and also the interactions between the two. The chief constraint at the moment on modelling sediment transport under waves and currents is the lack of a reliable theory for predicting sediment movement in wave dominated regions. The work should aid in the development of numerical models to predict the behaviour in estuaries under the action of both waves and currents.

1 INTRODUCTION

The movement of sediment in estuaries frequently causes problems. Any engineering works undertaken in an estuary may affect the pattern of sediment erosion or deposition and lead to beneficial or deleterious affects at locations throughout the estuary, occasionally at places remote from the site of the original engineering works. Thus the construction of docks, quays or jetties may lead to increased rates of sediment deposition or alternatively erosion. The maintenance or enlargement of existing shipping channels may have an impact on sediment movement and any alterations in fluvial flow or tidal storage may affect patterns of sediment erosion or deposition. Before embarking upon such engineering works it is therefore important for their impact on an estuary to be assessed to enable rational decisions to be made about the advisability of carrying out proposed schemes and to assess which schemes will achieve the required aims. Because of the complexity of the physical system representing an estuary recourse has frequently to be made to numerical models to make such predictions.

There has been a history of the successful use of numerical models to predict the behaviour of both cohesive and non-cohesive sediments in estuaries under the action of tidal currents (Abbott, 1979; Hydraulics Research Station 1982 and Verwey 1983). In these models, however, only the effects of currents on sediment transport is calculated and, at present, no allowance is made for the effect of waves. Though in most estuarial cases the movement of sediment is dominated by the currents, in certain circumstances waves may have a significant effect. Numerical models have been developed at HR for the calculation of waves (Southgate, 1981 and 1984). The physics of the interaction of waves and currents has already been described elsewhere (Longuet-Higgins, 1970; Noda, 1974 and Noda et al 1974). This report considers aspects of the problems of combining both current and wave numerical models to allow for these interactions and of determining any subsequent sediment movement. It is not intended to cover in this report either the numerical modelling of currents or waves separately or the physics of the interactions between the two as these have been described extensively elsewhere.

Though it is known that currents and waves interact and equations have been derived describing such interactions and also that waves affect the transport of sediment by currents, the relative magnitude of the various effects in practical situations remains unclear. A general assessment of their impact in common estuarial situations must await the development and implementation of the sort of numerical model discussed in this report.

2 SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS

Sediment transport in estuaries may depend upon both the waves and the currents. To simulate the movement of non-cohesive sediments in an estuary it is necessary to have a theory which will predict the local sediment transport rate. Many of the theories of sediment transport under waves and currents have been developed from theories of sediment transport under steady uni-directional currents. In the limit of zero waves these theories become identical to the equivalent, steady uni-directional flow equations which produce satisfactory predictions. The waves and currents theories can be regarded as extrapolations of the steady uni-directional flow theories. A comparison of such theories with field and laboratory data has indicated that, providing the effect of the current dominates that of the waves, they provide reasonable predictions but that when waves dominate the results are less satisfactory (Hydraulics Research, 1985). The predictions in the case where waves predominate could be improved if a reliable predictor were available for the case of waves alone. The main findings of the comparison are summarised in Appendix 1.

The sensitivity of sediment transport rate to both waves and currents is demonstrated by Figure 1 which shows the sediment transport rate calculated using Ackers and White-Swart equations. This sensitivity implies that to effectively include both waves and currents it is necessary to model both adequately.

Thus one is lead to the conclusion that to include the effects of sediment transport under waves and currents one should have a model which has both a flow component and a wave component.

When the waves and currents are not in the same direction consideration must also be given to the direction of the resulting sediment transport. The net movement of the sediment may not necessarily be in the direction of the waves or the current. Since the direction of the net shear may be different at different levels throughout the depth it is possible that the bed load may move in one direction and the suspended load may move in another.

Careful attention must also be given to the methods used to simulate the wave conditions. Under many conditions it can be assumed that waves behave linearly so that wave solutions may be superimposed. Since sediment transport is, in general, a non-linear process, however, it is invalid to superimpose solutions including sediment movement. It follows that the wave-field must be determined completely before any sediment transport is calculated. If waves of different directions are present this produces a complex moving pattern of shear stress distribution whose effect on sediment transport is unknown.

In applications the influence of bed slope may also have to be considered. Laboratory experiments have normally been limited to flat beds or beds sloping gently in the direction of the waves but in real situations the bed slope may influence the direction and amount of sediment transport.

3 DETERMINATION OF WAVES AND CURRENTS

Numerical models have been developed at HR for the calculation of waves in coastal situations and the calculation of tidal movements in estuaries. The only difficulty is in combining two such models and ensuring the appropriate interactions between the two.

Numerical modelling of short-period waves (2s to 15s) is normally performed using ray methods. These can allow for the effects of depth refraction, shoaling, reflections and diffraction (Southgate, 1981 and 1984). The area of interest is represented using a rectangular grid, typically each grid cell being of the order of 200m by 100m. Southgate (1984) described a technique of using a ray method combined with a spatial averaging technique whereby account can be taken of trends associated with a bundle of rays rather than the behaviour of an individual ray. The results obtained are in the form of an array of spot wave heights and phases covering the whole studied area. This method has the advantage that the resultant of two or more interacting wave trains may be calculated. Such ray methods essentially assume steady state conditions for the wave climate.

A range of tidal flow models are available at HR to simulate the diverse conditions that may occur in tidal problems. The finite-difference scheme used in such models may be implicit or explicit. In general explicit schemes involve fewer calculations per timestep but stability constraints limit the length of timestep that can be used. Implicit schemes can utilise larger timesteps but each timestep involves more computational effort. Explicit schemes can readily be adapted to take advantage of the DAP available at HR.

In combining the wave and current models there would be advantages if they could use the same numerical grid or if one was a subset of the other.

4 INTERACTIONS OF WAVES AND CURRENTS

Under certain circumstances the mutual interaction of waves and currents may be significant (Noda et al 1974). In these cases it is not sufficient to calculate the currents and waves independently. Even in the absence of tidal currents local variations in wave height result in wave-induced Reynolds' stresses or radiation stresses which drive currents (Longuet-Higgins, 1970 and Noda 1974). Thus in these situations the wave field must be determined first, the Reynolds stresses calculated and their effect included when determining the current field. A summary of the equations describing wave-induced Reynold's stresses is given in Appendix 3.

Variations in the spatial velocity distribution cause wave refraction. Thus where this is significant the current field must be known first and then the wave field may be calculated.

The magnitude of these current-wave interactions will vary with the situation and conditions and may frequently be insignificant. Initially some effort should be expended on characterising those conditions where such interactions may be ignored and those where it is important that they be included.

To include all these interactions it is necessary that the part of the model that calculates the wave refraction should be capable of including the effect of currents and that the flow part should be capable of including the Reynolds stresses induced by variations in wave height. If the waves and currents elements are separate in the model then some iterative technique is required in which the wave and current fields are solved for successively.

If the flow element is solved using a time-stepping procedure consideration should be given to the frequency with which the wave-field needs to be calculated. In many tidal situations, provided the waves are only weakly dependent upon the currents, the timescale for variations in the current field will be very much smaller than that for variations in the wave field. Consideration may then be given to whether different timesteps could be used for the calculation of the wave and current fields so that the wave field is up-dated less frequently than the currents.

5 EFFECT OF WAVES AND CURRENTS ON BED FEATURES

The presence of waves affects the bed features that are developed on a mobile bed and hence influences the hydraulic roughness; a knowledge of which is important in the determination of the flow field. Swart (1976) relates the hydraulic roughness to the bed form steepness though as he says this 'does not solve the problem of the determination of the bed roughness, it just shifts it'. Swart presents equations to predict bed form steepness and hence hydraulic roughness under wave dominated conditions but it is unclear how reliable such equations are.

6 MOVEMENT OF COHESIVE SEDIMENTS

In the current study we have restricted attention to the movement of non-cohesive sediments and have not considered cohesive sediments but the work that has been done does have some bearing upon the cohesive behaviour. The presence of waves leads to an increase in the maximum shear stress developed in the flow and this may lead to hindered settling of cohesive material in suspension or to the erosion of material from the bed if the magnitudes of the waves and currents are sufficiently large. Expressions for the bed shear stress under waves and currents are discussed in Appendix 2.

The presence of waves in a channel induces a non-zero mean velocity which, close to the bed, is in the direction of the waves. This can induce a steady motion in mud layers on the bed in the direction of the waves (Lhermitte, 1958 and Migniot, 1977). For muds of viscosity ranging from 3 x $10^{-5}m^2/s$ up to 7 x $10^{-5}m^2/s$ Lhermitte (1958) found the following expression for the drift velocity $\bar{u}_s$ at the surface of the mud bed,

$\bar{u}_s$ = 0.25 x maximum drift velocity in neighbourhood of bed

Migniot then quotes this as:

$\bar{u}_s = 0.25 \times 0.18\ u_o^{1.6}\ d^{-0.6}$ in cgs units though his expression for the maximum drift velocity is curious.

7 CONCLUSIONS

To include the effect of sediment transport under waves and currents in numerical estuary models the Ackers and White uni-directional sediment transport equations can be simply adapted by altering the effective shear. This should produce satisfactory predictions of transport rates provided the effect of the currents dominates that of the waves. The predictions will be less satisfactory where the waves dominate the currents. Since the sediment transport rate is sensitive to the wave height it is suggested that consideration should be given to the accuracy with which the wave field needs to be modelled and the possible effect of iteractions between the waves and currents.

The equations for the bed shear under waves and currents may also be used to include the effect of waves on the behaviour of cohesive sediments.

It is suggested that more work is directed at:

(a) a better expression for the bed shear under waves and currents,

(b) the direction of sediment transport when waves and currents are not in the same direction,

(c) the prediction of the hydraulic roughness of the bed under waves and currents,

(d) the development of a reliable theory for sediment transport under waves alone so that improvements could be made to predictions in the wave dominated area.

8 REFERENCES

Abbott M B, 1979, Computational hydraulics; Elements of the theory of free surface flows, Pitman.

Ackers P and White W R 1973, Sediment transport : new approach and analysis J Hydraul. Div. ASCE., 99, HY11, 2041-2060.

Bakker W T 1974, Sand concentration in an oscillatory flow, Chapter 66 Coastal Engineering.

Bijker E W 1967, Some considerations about scales for coastal models with moveable beds, Delft Hyd. Lab Publ. No 50.

Fredsoe J, 1983, The turbulent boundary layer in combined wave-current motion, Institute of Hydrodynamics and Hyd. Eng., Tech. Univ of Denmark, Report No 259.

Hydraulics Research Station, 1982, HRS Tidal modelling system, HR Report IT 238.

Hydraulics Research, 1985, Sediment transport under waves and currents, HR Report SR 22.

Jonsson I G, 1967, Wave boundary layers and friction factors, Coastal Engineering, Tokyo, pp 127-149.

Lhermitte, P, 1958, Contribution à l'étude de la couche limite des houles progressives, COEC No 136, Ministère de la Défense Nationale, Paris.

Longuet-Higgins M S, 1970, On the longshore currents generated by obliquely incident sea waves. J. Geophys Res 75.

Migniot, C, 1977, Action des courants de la houle et du vent sur les sédiments, La Houille Blanche, pp9-47.

Noda E K, 1974, Wave-induced nearshore circulation, J Geophy Res 79 pp 4097-4100.

Noda E K et al, 1974, Nearshore circulations under sea breeze conditions and wave-current interactions in the surf zone, Rep. TETRA-P-72-149-4, pp1-216, Tetra Tech, Pasadena, Calif.

Southgate H N, 1981, Ray methods for combined refraction and diffraction problems. Hydraulics Research Station Report, IT 214.

Southgate H N, 1984, Techniques of ray averaging, Int J. Num Methods in Fluids 4 pp 725-747.

Swart D H, 1976, Computation of longshore transport, Delft Hyd Lab Report R 968 Part 1.

Van de Graaff, J and Van Overeem J, 1979, Evaluation of sediment transport formulae in coastal engineering practice, Coastal Engineering, 3, pp 1-32.

Verwey A, 1983, Computational hydraulics in hydraulic design, in Developments in hydraulic engineering - edited by P Novak, Applied Science Publishers.

Willis D H, 1978, Sediment load under waves and currents. Proc. 16th Coastal Engineering Conference, Hamburg, 2, pp 1626-1637.

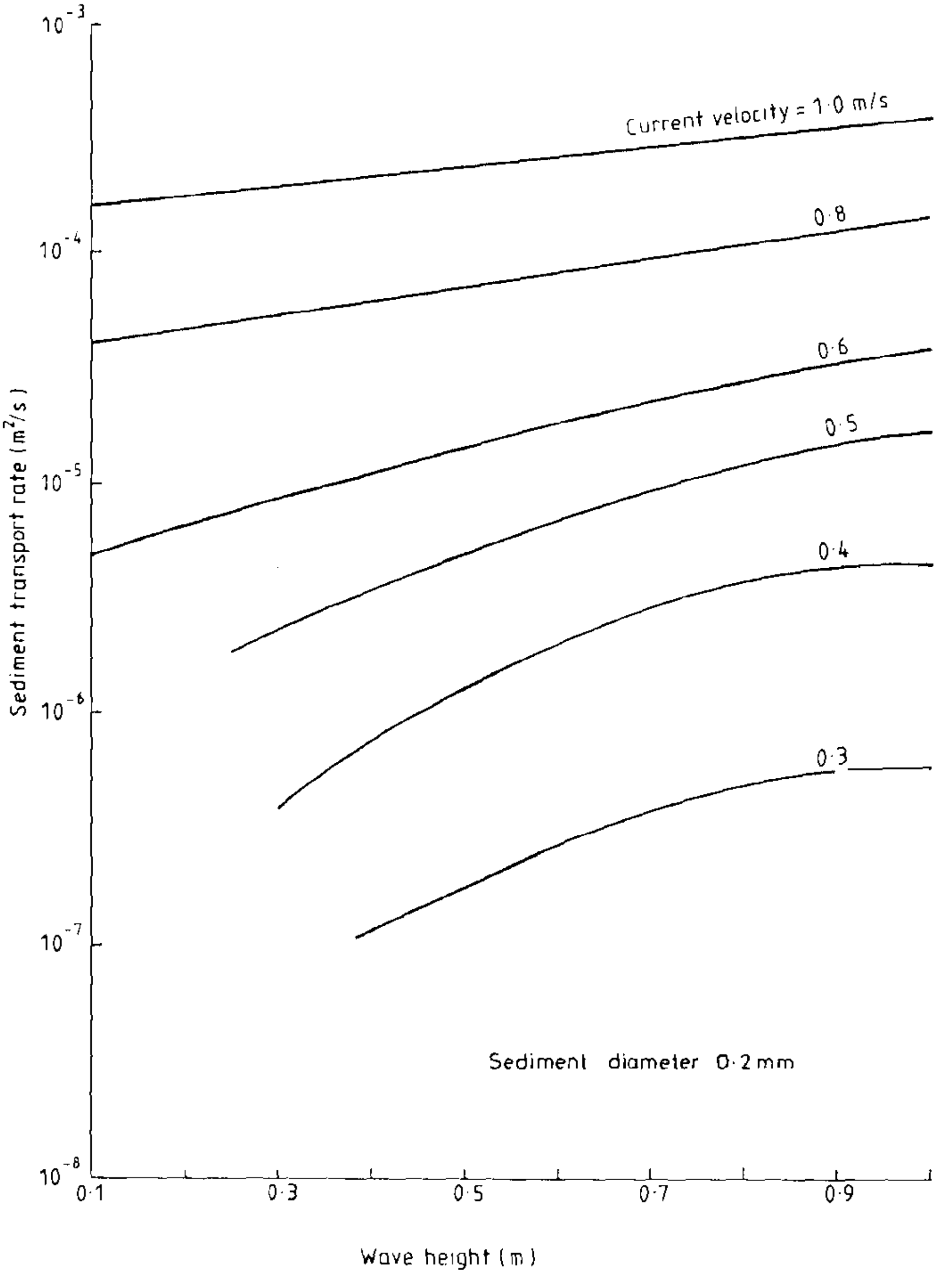

Fig 1 Sediment transport rate against wave height

APPENDIX 1

SEDIMENT TRANSPORT UNDER WAVES AND CURRENTS

The following theories have been compared with field and flume observations:

1. Frijlink-Bijker
2. Ackers and White-Swart
3. Ackers and White-Willis
4. Ackers and White-van de Graaf and van Overeem
5. Engelund and Hansen-Swart

The above sediment transport equations have been derived from equations from uni-directional sediment transport. As an illustration of the methods involved we describe the Ackers and White-Swart method.

Ackers and White-Swart

The Ackers and White equations for sediment transport utilise four parameters, n,A,m and C which depend upon the dimensionless grain size D_{gr} of the sediment. Swart keeps these unchanged when applying the equations to sediment transport under waves and currents. The mobility in the case of currents above is defined by

$$F_{gr} = \frac{{v_*}^n}{\sqrt{(gD(s-1))}} \left[\frac{V}{\sqrt{(32)}\log_{10}(10d/D)}\right]^{1-n} \qquad (A1.1)$$

and Swart defines the corresponding variable for waves and currents, ${F_{gr}}^{wc}$ by:

$$ {F_{gr}}^{wc} = F_{gr} \times \left(\frac{v_{*wc}}{v_{*c}}\right)^n, \qquad (A1.2)$$

where $v_{*wc} = v_{*c} \left(1+\left(\xi \frac{u_o}{V}\right)^2\right)^{\frac{1}{2}}$ (see Appendix 2 for notation)

The equation for sediment concentration in the case of currents alone is

$$X = G_{gr} \, s \, \frac{D}{d} \left(\frac{V}{v_*}\right)^n \qquad (A1.3)$$

In the case of currents and waves Swart replaces this by

$$X = G_{gr} \, s \, \frac{D}{d} \left(\frac{V}{v_{*wc}}\right)^n \qquad (A1.4)$$

Thus, denoting v_{*wc}/v_{*c} by β the equation for the sediment concentration becomes

$$X = C \left(\frac{F_{gr}}{A} \beta^n - 1\right)^m \frac{sD}{d} \left(\frac{V}{v_{*c}}\right)^n \frac{1}{\beta^n} \tag{A1.5}$$

The comparisons with observations indicated that, in general, the methods performed satisfactorily when the effects of the currents dominated those of the waves but that they uniformly over-predicted the transport rate when waves dominated, Fig 2. This was in part due to the expression adopted by the various methods for the shear stress under waves and currents. The comparisons seemed to indicate that the predictions by the various theories depended very heavily on the expression used for the bed shear stress developed under waves and currents, v_{*wc}. When equation for bed shear stress (A2.6) was replaced by equation (A2.3) then improved predictions resulted though transport rates were still over-predicted, Figs A1 and A2. An ad-hoc correction to the shear stress was employed which improved the predictions.

In general the methods based on Ackers and White sediment transport theory performed better than the others. Within the group there were variations in the behaviour of the different adaptions but, taking into account the inadequacies of the data upon which these methods were tested, it would be difficult to argue that these variations were significant.

It is unlikely that predictions in the wave dominated region can be significantly improved until more reliable equations are available for sediment transport under waves alone.

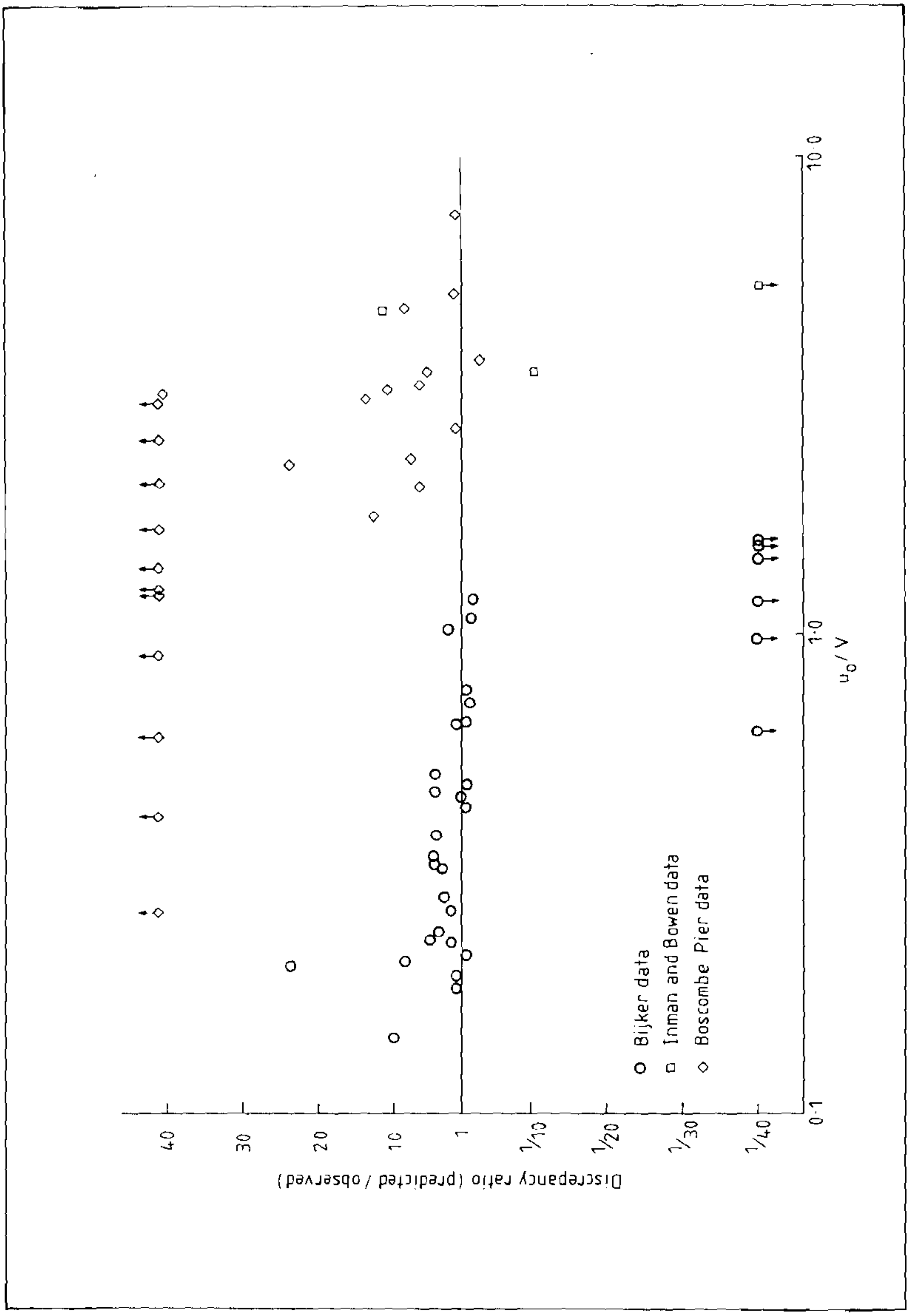

Fig A1. Discrepancy ratio against u_0/ V, Ackers and White - Swart

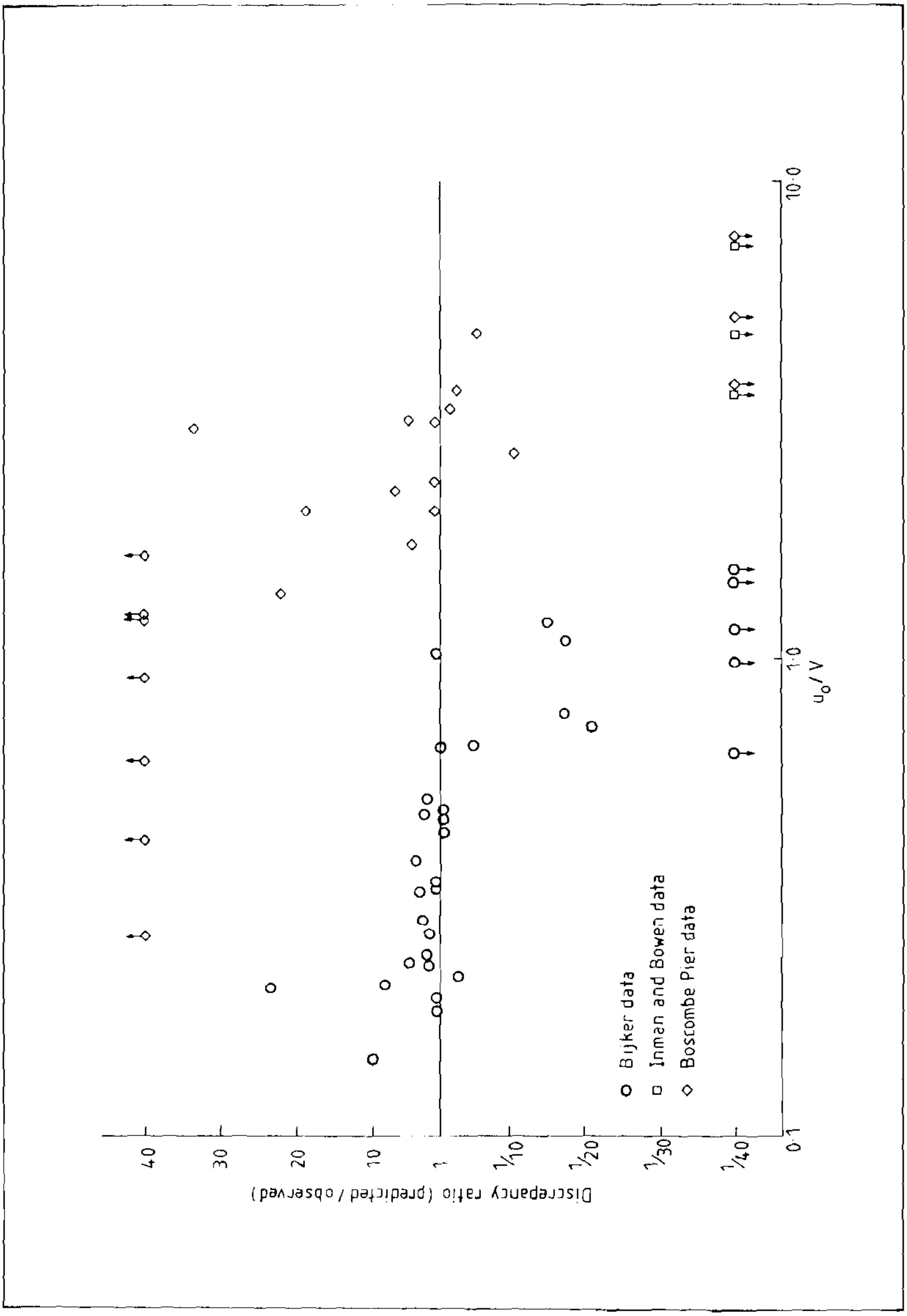

Fig A2 Discrepancy ratio against u_0/V, Ackers and White-Swart with Bijker shear stress integration

APPENDIX 2

SHEAR STRESS UNDER WAVES AND CURRENTS

Bijker (1967) assumed that the velocities under waves and currents were the sum of a uniform flow logarithmic velocity profile,

$$v(y) = \frac{v_\star}{\kappa} \ln \frac{y}{d} , \qquad \text{(A2.1)}$$

and the wave velocity given by the first-order wave theory

$$u_o = \frac{\omega H}{2 \sinh kd} \qquad \text{(A2.2)}$$

where H is the wave height, d is the depth and k is the wave number. Bijker then calcualted the mean component of the resultant bed shear in the direction of the current to give

$$\frac{\tau_{wc}}{\tau_c} = \frac{2}{T} \int_{-T/4}^{T/4} [(1+\xi \frac{u_o}{V} \sin\omega t \sin \phi) \sqrt{}(1+\xi^2 \frac{u_o{}^2}{V^2} \sin^2 \omega t + 2\xi \frac{u_o}{V} \sin\omega t \sin \phi))dt \qquad \text{(A2.3)}$$

where

$$\xi = \frac{p \kappa C_h}{g^{\frac{1}{2}}}, \qquad T \text{ is the wave period,} \qquad \text{(A2.4)}$$

$p = 0.45$, C_h is the Chezy roughness coefficient given by $C_h = 18 \log_{10} \frac{12d}{r}$, r is the bed roughness and ϕ is the angle between the wave crests and the normal to the current. Bijker evaluated the elliptic integral in (A2.3) for a range of values of ϕ, u_o and V and fitted an approximation to (A2.3) of the form

$$\frac{\tau_{wc}}{\tau_c} = a + b \, (\xi \frac{u_o}{V})^c \qquad \text{(A2.5)}$$

for various values of ϕ, see Bijker (1967) for details. These approximations are only valid in particular parameter ranges and should only be used in those ranges. In the case where the component of the shear in the direction of the current is always positive then (A2.3) reduces to

$$\frac{\tau_{wc}}{\tau_c} = 1 + \tfrac{1}{2} \, (\xi \frac{u_o}{V})^2 \qquad \text{(A2.6)}$$

but this should only be used if $u_o < V$.

The maximum bed shear stress is given by

$$\left(\frac{\tau_{wc}}{\tau_c}\right)_{max} = 1 + \left(\xi \frac{u_o}{V}\right)^2 + 2 \xi \frac{u_o}{V} \sin \phi \qquad \text{(A2.7)}$$

Swart (1976) suggested replacing equation (A2.4) by the equation

$$\xi = C_h \left(\frac{f_w}{2g}\right)^{\frac{1}{2}} \qquad \text{(A2.8)}$$

where f_w is the Jonsson wave friction factor which can be approximated by

$$f_w = \begin{cases} \exp\left(-5.98 + 5.21 \left(\frac{a_o}{r}\right)^{-0.19}\right) & \frac{a_o}{r} > 1.57 \\ 0.3 & \frac{a_o}{r} < 1.57 \end{cases} \qquad \text{(A2.9)}$$

where a_o is the orbital amphitude at the bed, that is

$$a_o = \frac{Tu_o}{2\pi} \qquad \text{(A2.10)}$$

Bijker's model, using a logarithmic velocity profile for the current and a first order wave theory, is relatively crude and must be seen as a major weakness in any theory of sediment transport that uses it.

There are more sophisticated models but these frequently involve solving a differential equation through the depth to obtain the flow structure and hence obtaining the shear stress (Bakker, 1974 and Fredsoe, 1983). Such models would be computationally expensive to implement. The possibility arises, however, of developing appropriate 'look-up' tables which might lead to savings in computation.

APPENDIX 3

WAVE INDUCED RADIATION STRESS

Variations in wave height induce radiation stresses given by:

$$M_x = -\frac{1}{\rho(\eta+d)}\left(\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y}\right) \qquad \text{(A3.1)}$$

$$M_y = -\frac{1}{\rho(\eta+d)}\left(\frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\tau_{yx}}{\partial x}\right) \qquad \text{(A3.2)}$$

where η is the local height of the water surface above the mean water level

$$\sigma_{xx} = \frac{1}{8}\rho g H^2\left[(2n-\tfrac{1}{2})\cos^2\theta + (n-\tfrac{1}{2})\sin^2\theta\right] \qquad \text{(A3.3)}$$

$$\sigma_{yy} = \frac{1}{8}\rho g H^2\left[(2n-\tfrac{1}{2})\sin^2\theta + (n-\tfrac{1}{2})\cos^2\theta\right] \qquad \text{(A3.4)}$$

and $\tau_{xy} = \tau_{yx} = \frac{1}{16}\rho g H^2 n \sin 2\theta$ (A3.5)

θ is the angle between the normal to the wave crest and the x-axis and

$$n = \tfrac{1}{2} + \frac{kd}{\sinh 2kd} \qquad \text{(A3.6)}$$

If we use the shallow water approximation equations (A3.3), (A3.4) and (A3.5) become

$$\sigma_{xx} = \frac{1}{16}\rho g H^2\left[3\cos^2\theta + \sin^2\theta\right] \qquad \text{(A3.7)}$$

$$\sigma_{yy} = \frac{1}{16}\rho g H^2\left[3\sin^2\theta + \cos^2\theta\right] \qquad \text{(A3.8)}$$

and $\tau_{xy} = \tau_{yx} = \frac{1}{16}\rho g H^2 \sin 2\theta$ (A3.9)

(see Longuet-Higgins 1970 and Noda, 1974 for details).

NOTATION

A		Ackers and White parameter
a		constant
a_o	m	wave rbital amplitude at the bed
b		constant
C		Ackers and White parameter
C_h		Chezy coefficient
c		constant
D	m	sediment diameter
d	m	flow depth
F_{gr}		sediment mobility, Ackers and White
F_{gr}^{wc}		sediment mobility under waves and currents
f_w		Jonsson friction factor
g	m^2/s	acceleration due to gravity
G_{gr}		dimensionless sediment transport rate
H	m	wave amplitude
k	m^{-1}	wave number, 2π/wavelength
m		Ackers and White parameter
n		Ackers and White parameter
p		constant
r	m	roughness of bed
s		specific gravity of sediment ρ_s/ρ
T	s	period of waves
u_o	m/s	maximum orbital velocity of waves in neighbourhood of the bed
$\bar{u}_s$	m/s	drift velocity at surface of mud bed

V m/s	mean current velocity
v_* m/s	shear velocity
v_{*c} m/s	shear velocity due to current
v_{*wc} m/s	shear velocity due to waves and currents
X	sediment concentration
y m	depth
β	ratio of shear velocity under waves and currents to shear velocity under currents (= v_{*wc}/v_{*c})
κ	Van Karman's constant
ξ	$\frac{p\kappa C_n}{g^{\frac{1}{2}}}$
ρ kgm^{-3}	density of water
ρ_s kgm^{-3}	density of sediment
τ_c $kgm^{-1}s^{-2}$	shear stress under currents
τ_{wc} $kgm^{-1}s^{-2}$	shear stress under waves and currents
ϕ	angle between the wave crests and the normal to the currents
ω s^{-1}	frequency