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INFRAGRAVITY WAVES AND HARBOUR DESIGN

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ABSTRACT

The significance of infragravity waves (periods 20 seconds to many minutes) for harbour design is explained. The two basic types of infragravity motion associated with wave grouping, set-down and surf beat, are identified.

A theoretical model has been developed for disturbances forced by wave grouping to allow for realistic seabed slopes. This extends an already validated model of set-down which applies for a flat seabed and for intermediate water depths typical of harbour entrances. It is shown that a forced disturbance occurs, in addition to set-down, when the slope of the seabed is taken into account.

The model has been applied to long wave data collected off the entrances to five harbours: Port Talbot, Dover, Shoreham, Sunderland and Barrow-in-Furness. This has enabled the amount of surf beat energy in the measured data to be quantified for the five sites. As a result it is shown that surf beat as well as set-down is important in the spectrum of long period energy at harbour entrances.

Unlike set-down, there is no adequate theory for surf beat which enables its magnitude to be determined without site measurements. The analysis of measurements presented here indicates that an important component of surf beat consists of free long waves propagating in a seaward direction. Their counter-part in physical models of harbours will be re-reflected towards the model harbour by the wave-maker, giving rise to an unrealistic representation of surf beats, unless the surf beats can be absorbed at the wave-maker. Recommendations are made for research to overcome this problem and at the same time improve our understanding of surf beats.

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1 INTRODUCTION

Moored ships behave like masses on springs, with the elasticity of the moorings determining the effective spring constant. As a result, ships on their moorings possess resonant, or natural, periods which are proportional to the square root of the ratio given by the effective mass over the effective spring constant. Thus, the larger the vessel and the softer the moorings, the longer the natural periods. To avoid excessive mooring loads due to resonance with the ordinary waves it is clear that moorings for vessels need to be chosen so that the natural periods are longer than the wave period and, typically, these periods vary from some 20 seconds, for vessels of coaster size, to two minutes or so for large tankers. This means resonance with the ordinary waves is generally avoided. It is now realised, however, that energy in the sea also exists at periods of 20 seconds to many minutes. The ability of this longer period energy to excite resonant responses of moored ships, and hence cause high mooring loads, has been amply demonstrated in many random wave physical model investigations of harbours carried out at Hydraulics Research (HR). The few full scale measurements of moored ship motions in harbours that have been carried out also show that horizontal vessel excursions occur at periods longer than the wave period (a comparison of model and full scale moored ship motions is presented in Ref 1).

The full significance of long period energy for harbour design has really only been appreciated within the last twenty years. An important factor has been the change from using single period, or regular, waves in physical models to the use of more realistic random, or irregular, waves. In contrast to regular waves, random waves reproduce the wave grouping present in the real sea and research has shown that this characteristic leads, through non-linearities in the ordinary wave motion, to long period disturbances at wave group periods. The spectrum of group periods covers the range from about 20 seconds up to many minutes, ie the range of typical moored ship resonances. Prior to this research, some harbours were known to have mooring problems due to long waves but the source of long wave energy was unknown and it was thought that only certain sites in the world would be affected. But it is now clear that all harbours exposed to waves will automatically experience some long wave energy associated with grouping of the ordinary waves. Whether the long waves are significant enough to cause mooring difficulties at a site proposed for development then becomes one of the factors that requires study.

The long period energy produced by wave grouping falls within the "medium frequency" range extending from periods of about ½ minute to 3 hours. This energy lies between wind generated waves and tides. Such waves are also termed "infragravity". It can be seen there is no standard terminology on this yet. Here we use the term infragravity to describe all the long period energy associated with wave grouping. The energy can be broadly classified under the two headings "set-down beneath wave groups" (or just set-down) and "surf beats".

1.1 Set-down beneath wave groups

Starting with groups of waves propagating towards the shore, long period disturbances are generated by non-linearities in the ordinary wave motion. These disturbances are sometimes called "bound long waves" because they are bound to groups of incoming waves and so propagate at the group velocity. But they are not true long waves. For example their speed of propagation, the group velocity, is less than the phase speed of a true long wave at the same period. For this reason the term set-down beneath wave groups is used here since it avoids referring to the disturbance as a wave. The term set-down is also more descriptive as it indicates that a trough develops beneath groups of large waves: a feature of the disturbance in question.

Longuet-Higgins (Ref 2) used a Stokes' expansion of the wave equations to describe set-down and that theory has been validated using long crested random or irregular waves at a number of laboratories including HR (Ref 3, 4, 5, 6, 7) for deep water and intermediate depths typical of harbour entrances. However, the validation of the theory for set-down applies to a flat bed and the applicability of the theory for sloping seabeds, the more realistic situation, is explored in Section 2 of this report. Evidence of set-down in real sea measurement has also been obtained by HR at a number of sites and this aspect is discussed in more detail in Section 3 of this report.

The difficulty with the theory used by Longuet-Higgins (Ref 2) is that in shallow water it predicts set-downs that exceed the amplitudes of the primary waves. This unrealistic behaviour is caused by the breakdown of the Stokes' expansion of the wave equations in shallow water. This breakdown occurs for water depths well offshore of the surf or breaker zone and it means there is no theory for set-down in the region extending out from the surf zone to intermediate water depths. Nevertheless, field measurement (Refs 8, 9) have indicated that set-down is reflected in the surf zone to give rise to free (of wave groups) seaward going long waves. This type of long period energy is considered under the heading of surf beats.

1.2 Surf beats

This, the second broad heading for long period energy, is chosen here to encompass all the free long waves, ie all the energy at wave group periods apart from set-down. Surf beats are not well understood and there is no accepted theory that can be used to predict their magnitude. However, a number of mechanisms have been proposed to explain their generation and these are considered in more detail in Section 4. Here we consider the likely sources of energy for these waves.

One source which has already been mentioned, is the incoming set-down itself. Due to its long wave character it appears unlikely that the energy in set-down is absorbed at the shoreline. This contrasts with the primary waves which, typically, dissipate their energy via breaking in the surf zone. Long period waves tend to reflect even from beaches which suggests that set-down will produce free long waves in the vicinity of the surf zone once the primary waves are dissipated. There is support for this idea in field data. Tucker (Ref 9) observed that long waves measured by a recorder offshore of a beach had a significant negative correlation with groups of large waves. This correlation occurred with a lag corresponding the time taken for groups of waves, and therefore set-down, to travel to the surf zone plus the time for a long wave to travel back to the recorder (Ref 2). The negative correlation and the time lag provide the link between these long waves and set-down.

The other possible source of energy for surf beats is associated with the set-up in mean water level that occurs in the surf zone due to wave breaking. As the waves making up a group enter shallow water they will tend to break in varying depths with higher waves breaking in deeper water. This in turn will tend to produce a set-up inside the surf zone that fluctuates with the wave groups. An attempt to describe these fluctuations has been made by Symonds et al (Ref 10) for a simplified situation. This theory indicates that the oscillation of the break point of the waves back and forth on a beach slope will produce both a free long wave propagating back out to sea as well as a shoreward going long wave. The shoreward going wave will be largely reflected at the shoreline to produce a second seaward going component. The resultant of the two seaward going long waves then forms the surf beat.

At this stage the relative importance of these two sources of surf beat energy, ie the reflection of set-down and the oscillating break point theory, is unclear. Due to the complexity of real sea data in the vicinity of the surf zone, eg the directional spread of wave energy that remains even after wave refraction and the strong non-linearity of breaking waves, it has not been possible to provide conclusive evidence for either theory. It seems clear that simplified numerical and physical model experiments are needed to establish the relevant physics.

As described so far, surf beats are seaward going long waves. However, the uneven seabed topography will tend to refract some of these waves to such a degree that they remain trapped by the coastline. These trapped waves are called edge waves. They have amplitudes that decay with distance offshore and they propagate parallel to the coastline. Such waves, with periods in the infragravity range, have been identified in site data (Ref 11).

Under the terminology being used in this report, surf beats encompass both seaward going long waves and edge waves trapped by the coastline and we use the term infragravity to describe surf beats plus set-down. It should be noted, though, that some authors include set-down under the surf beat heading. Again, there is no agreed terminology for these disturbances.

1.3 Significance for harbour design

As already mentioned, set-down for a flat seabed has been adequately described for water depths typical of harbour entrances with both theory and laboratory data agreeing on its magnitude for long crested waves. Although not a true long wave Bowers (Ref 12) has shown, both theoretically and in laboratory tests, that set-down will excite long waves inside harbours. Thus, set-down retains the character of a long wave in that it excites a residual disturbance within harbours that can cause ships to resonate on their moorings even though the primary waves are much reduced by protective breakwaters. Many random wave physical models have demonstrated this behaviour with berth "downtime" being largely controlled by long period vessel responses. The agreement between long crested wave physical models and theory on both the magnitude of set-down and its ability to excite long waves inside harbours, shows that the residual long wave disturbance present in these models, due to the incident set down, is a realistic effect. Theory also indicates that set-down is sensitive to the amount of short-crestedness in sea waves and under the present research contract is is planned to build and validate a physical model of a harbour using a short crested wave-maker (Ref 13). This should ensure an even more realistic representation of set-down in physical models which, in turn, can be expected to lead to even more accurate estimates of berth downtime.

Surf beats, being free long waves, will also excite residual long waves inside harbours which in turn could cause moored ship resonances. It is important, therefore, that surf beats as well as set-downs are well represented in harbour modelling. Flume tests do indicate that surf beats will develop in physical models (Refs 3, 6). But the absence of a validated theory for surf beats makes it difficult to check the magnitudes of these model disturbances. A technique for separation of set-down and surf beat components in the long period spectrum has been developed by HR for site data (Ref 14). It is hoped to apply this analysis method to data collected at the harbour site due to be modelled (using short crested waves) under the present research contract. A comparison of the surf beat components in the model with those in the site data will then form a check on the ability of the physical model to reproduce surf beats.

In this report the effect of typical seabed slopes on the flat bed theory for set-down is investigated in Section 2. Site data on infragravity waves gathered by HR in past project work is assessed in Section 3 with a view to identifying surf beat energy. Finally, mechanisms for surf beat generation are reviewed in Section 4.

2 SET-DOWN ON A SLOPING SEABED

The slope of the seabed can be defined by a length scale that, typically, exceeds the wavelengths present in the sea. This means an adequate description of ordinary sea waves on a sloping seabed can be obtained by assuming the waves have time to adjust their amplitudes and wavelengths to a changing water depth such that they still satisfy the normal dispersion relation for a flat bed, ie

$$\omega^2 = kg \tanh kh. \qquad (1)$$

Here, ω is the radian frequency corresponding to wave period T,

$$\omega = \frac{2\pi}{T},$$

g is the gravitational constant, k is the wave number corresponding to wavelength λ, $k = \frac{2\Pi}{\lambda}$
h is the water depth.

As waves propagate into shallower water (decreasing h) we can assume the period and, therefore, frequency remain constant. Equation (1) shows that k must increase, ie wavelength decreases. This reduces the phase velocity of the waves which in turn causes refraction over a varying seabed topography. Equations that describe the resulting changes in wave direction and amplitude (these occur on the length scale representative of depth changes) are much used in wave modelling. They allow, for example, the transformation of a deep water wave spectrum into the spectrum at a nearshore point of interest (Refs 15, 16) for a quite general seabed topography.

The success of this assumption, concerning wave propagation over a varying seabed, suggests that a similar assumption may hold good for set-down beneath wave groups. This would then enable the already validated flat bed theory for set-down to be used on a varying seabed. This assumption, however, is more likely to break down for set-down as the wavelengths involved are of the order of kilometers instead of the hundreds of metres for the ordinary waves. The long wavelengths of set-down are, therefore, not all that different from typical length scales for water depth changes. It is this matter that is now investigated.

2.1 Theoretical model

For convenience, we represent the seabed by straight parallel contours running perpendicular to the x axis of a right-handed orthogonal co-ordinate system (Fig 1). The still water surface is the plane $z = o$. As set-down is sensitive to the directional spread of energy in the wave spectrum we allow for a multi-directional or short crested sea approaching from offshore with its mean direction along Ox. Initially, we allow for an unspecified depth variation $h(x)$ in working out general expressions for set-down and then, later, for investigation of specific cases we assume a constant seabed slope α ie

$$h = x\tan\alpha. \qquad (2)$$

Surface waves are described by irrotational motion in which the fluid velocity vector $\underset{\sim}{q}$ is derived from a velocity potential ϕ,

$$\underset{\sim}{q} = (u, v, w) = -\underset{\sim}{\nabla}\phi.$$

Incompressibility leads to the basic wave equation,

$$\nabla^2\phi = 0 \qquad (3)$$

Solutions to (3) are sought subject to the following boundary conditions.

On the seabed, $z = -h$, the normal velocity vanishes.

$$u\frac{dh}{dx} + w = 0 \tag{4}$$

on the free surface, $z = \eta$, we have the kinetic condition,

$$\frac{\partial \eta}{\partial t} + u\,\frac{\partial \eta}{on} + v\,\frac{\partial \eta}{oy} - w = 0, \tag{5}$$

and Bernoulli's equation, taking account of constant air pressure at the surface,

$$\frac{1}{2}q^2 + g\eta - \frac{\partial \phi}{\partial t} = 0 \tag{6}$$

The above system of equations is non-linear with product terms in (5) and (6) making it difficult to obtain exact solutions. As already explained, we are interested in examining the validity of the flat bed solution for set-down on a sloping seabed in intermediate depths. We therefore make the same assumption as in the flat bed solution and carry out a Stokes' expansion of the wave quantities in powers of the wave amplitude.

2.2 First order solution

To first order we assume the product terms in (5) and (6) are smaller than the other terms in the equations. Then it is relatively straightforward to solve the resulting linear system of equations. We denote the first order velocity potential and surface elevation by $\phi^{(1)}$ and $\eta^{(1)}$, respectively, and first of all we solve the equations on a flat bed ie put

$$\frac{dh}{dx} = 0$$

The spectrum of set-down is built of pairs of primary waves. We therefore take two primary wave components with frequencies ω_1 and ω_2 propagating at angles Θ_n and Θ_m, respectively, with the x axis (see also Fig 1). The surface elevation at position $\underline{r}$ then takes the form,

$$\eta^{(1)} = a_{n2} \cos(\omega_2 t + \underline{k}_{n2}.\underline{r} + \epsilon_{n2}) + a_{m1} \cos(\omega_1 t + \underline{k}_{m1}.\underline{r} + \epsilon_{m1}) \qquad (7)$$

where

$$k_{n2}.r = k_2 \cos \Theta_n x + k_2 \sin\Theta\, y,$$

$$\underline{k}_{m1}.\underline{r} = k_1 \cos \Theta_m x + k_1 \sin\Theta_m y.$$

Also a_{n2}, a_{m1} are the amplitudes of the two components and ϵ_{n2}, ϵ_{m1} are their random phases. Solving the first order (linear) system of equations, we find each primary wave component satisfies the dispersion relation (1) and that the velocity potential takes the form,

$$\phi^{(1)} = \frac{a_{n2} g}{\omega_2} \frac{\cosh k_2 (z + h)}{\cosh k_2 h} \mathrm{Sin}(\omega_2 t + \underline{k}_{n2}.r + \epsilon_{n2}) + \frac{a_{m1} g \cosh k_1 (z + h)}{\omega_1 \cosh k_1 h} \mathrm{Sin}(\omega_1 t + \underline{k}_{m1}.\underline{r} + \epsilon_{m1}) \qquad (8)$$

Each component contributes an amount of energy to the wave spectrum $s_d(f,\Theta)$ defined by,

$$a_{n2}^2 = 2\ S_d(f_2,\Theta_n)\ df d\Theta, \qquad (9)$$

$$a_{m1}^2 = 2\ S_d(f_1\Theta_m) df d\Theta \qquad (10)$$

These equations link the wave amplitudes to spectral densities at the component frequencies and directions. The full short crested sea is then defined by the superposition of all these components and the sea surface is said to have the two dimensional spectrum $S_d(f,\Theta)$ which is a function of wave frequency and direction.

To encompass a varying water depth h(x) we must allow the amplitudes, wavelengths and directions of the spectral components to vary with x as well. As already explained, this can be done while retaining the flat bed relationships (1) and (7) to (10) since, typically, seabeds are relatively flat and the waves have time to adjust to the slowly changing depth. However, it can be shown that depth variations lead to the following additional requirement on each component in the spectrum,

$$\frac{d}{dx}(a^2 C_g \cos\Theta) = 0 \qquad (11)$$

Here, C_g is the group velocity or speed of propagation of wave energy,

$$C_g = \frac{\omega}{2k}\left(1 + \frac{2kh}{\text{Sinh}2kh}\right) \qquad (12)$$

Equation (11) expresses conservation of wave energy as waves refract over the varying seabed.

2.3 Second Order Solution

It is only necessary to go to second order in the Stokes' expansion of the wave equations to describe set-down beneath wave groups. We denote second order quantities with a superfix (2). Solutions to Laplace's equation (3) are sought for $\phi^{(2)}$ subject to the boundary condition (4) on the seabed and the following boundary condition on the surface z = 0.

$$\frac{\partial^2 \phi^{(2)}}{\partial t^2} + g\frac{\partial \phi^{(2)}}{\partial z} = \eta^{(1)}\left(\frac{\partial^2 w^{(1)}}{\partial t^2} + g\frac{\partial w^{(1)}}{\partial z}\right) + 2\,\underline{q}^{(1)}\frac{\partial \underline{q}^{(1)}}{\partial t} \qquad (13)$$

Equation (13) shows that a surface perturbation, on the right-hand side, forces a non-linear correction $\phi^{(2)}$ to the wave motion. This condition is obtained by Taylor expanding w and $\frac{\partial \phi}{\partial t}$ in (5) and (6), respectively, about z = 0 and then substituting for the second order surface elevation $\eta(2)$, from (6), into (5) and rearranging terms on the right-hand side.

The right-hand side of (13) can be evaluated by using (7) and (8) to define first order quantities. The products of these first order quantities lead to terms containing sum and difference frequencies $\omega_2 \pm \omega_1$. It is the fluctuations at the difference frequency that concern us here. They lead to set-down beneath wave groups.

Following the pattern of solution outlined in Section 2.2 for the ordinary waves, we calculate set-down first of all neglecting depth variations ($\frac{dh}{dx} = 0$) and then consider the effect of variable depth as a perturbation on this solution. Therefore, solving

(3) subject to (4) with $\frac{dh}{dx} = 0$ on the seabed, and subject to (13) on the surface, we obtain the usual flat bed solution for the set-down velocity potential $\emptyset_o^{(2)}$. Here, we use the suffix o to show that we have ignored depth variations.

$$\emptyset_o^{(2)} = A_{m,n} \cosh[|\underline{k}_{n2} - \underline{k}_{m1}|(z+h)] \text{Sin}(\omega^{-} t + (\underline{k}_{n2} - \underline{k}_{m1})\underline{r} + \epsilon_{n2} - \epsilon_{m1}) \tag{14}$$

where,

$$\omega^{-} = \omega_2 - \omega_1,$$

$$A_{m,n} = \frac{1}{2} g^2 a_{n2} a_{m1} \left[\frac{e^{-} + \frac{2k_1 k_2}{\omega_1 \omega_2} \omega^{-} [\cos(\Theta_n - \Theta_m) + \tanh k_1 h \tanh k_2 h]}{(\omega^{-})^2 \cosh|\underline{k}_{n2} - \underline{k}_{m1}|h + g|\underline{k}_{n2} - \underline{k}_{m1}| \text{Sinh}|\underline{k}_{n2} - \underline{k}_{m1}|} \right]$$

$$e^{-} = \frac{k_2^2}{\omega_2 \cosh^2 k_2 h} - \frac{k_1^2}{\omega_1 \cosh^2 k_1 h}$$

To find the perturbations $\emptyset_1^{(2)}$ to this solution due to depth variations we consider solutions to (3) of the following form,

$$\emptyset_1^{(2)} = G_{m,n}(x,z) \cos(\omega^{-} t + (\underline{k}_{n2} - \underline{k}_{n1}) \cdot \underline{r} + \epsilon_{n2} - \epsilon_{m1}), \tag{15}$$

where,

But this is not a solution to $\nabla^2 \phi = 0$

$$G_{m,n} = \sinh|\underline{k}^{-}|(z+h) \; (B_{m,n} + z E_{m,n})$$

$$+ \cosh|\underline{k}^{-}|(z+h) \; [C_{m,n} + z D_{m,n} + z^2 F_{mn}],$$

$$|\underline{k}^-| = |\underline{k}_{n2} - \underline{k}_{m1}|,$$

$$2|\underline{k}^-| F_{m,n} = - k_x^- \frac{d}{dx} |\underline{k}^-| A_{m,n},$$

$$|\underline{k}^-| D_{m,n} = - k_x^- \frac{d}{dx} |\underline{k}^-| A_{m,n},$$

$$2|k^-| E_{m,n} = -\frac{1}{A_{m,n}} \frac{d}{dx} (k_x^- A^2_{m,n}) + \frac{k_x^-}{|k|} \frac{d}{dx}|k^-| A_{m,n},$$

$$k_x^- = k_2 \cos\Theta_n - k_1 \cos\Theta_m.$$

The boundary condition (4) gives an expression for $B_{m,n}$,

$$|\underline{k}^-| B_{mn} = \frac{-h}{2A_{m,n}} \frac{d}{dx} (k_x^- A^2_{mn}) + \frac{k_x^- h}{2|\underline{k}^-|} \frac{d}{dx} |\underline{k}^-| A_{m,n}$$

The boundary condition (13) defines $C_{m,n}$,

$$C_{m,n} [g|\underline{k}^-| \sinh|\underline{k}^-|h - (\omega^-)^2 \cosh|\underline{k}^-|h] + gD_{m,n} \cosh|\underline{k}^-|h$$
$$+ gE_{m,n} \sinh|\underline{k}^-|h + B_{m,n} [g|\underline{k}^-| \cosh|\underline{k}^-|h - (\omega^-)^2 \sinh|\underline{k}^-|h] = H_{m,n}$$

where $H_{m,n}$ is the term on the right-hand side of (13) proportional to $\cos(\omega^- t + (\underline{k}_{n2} - \underline{k}_{m1}) \cdot \underline{r} + \epsilon_{n2} - \epsilon_{m1})$.

The above set of equations define, quite generally, the correction to the flat bed velocity potential for set-down. It can be shown that in deep water these corrections are relatively unimportant so we now consider the corrections in the limit of shallow water. In application we are interested in pressures on the seabed (see next Section dealing with real sea pressure sensor measurements). In the shallow water limit we find after some algebra, the bed pressure corrections $P_c^{(2)}$ to flat bed set-down take the form,

$$P_c^2 / \rho g = - \bar{\omega} I_{m,n} \sin (\bar{\omega} t + (\underline{k}_{n2} - \underline{k}_{m1}) \cdot \underline{r} + \epsilon_{n2} - \epsilon_{m1}), \tag{16}$$

where,

$$I_{m,n} [gh |\underline{k}^-|^2 - (\bar{\omega})^2] = \frac{1}{J_{m,n}} \frac{d}{dx} [J^2_{m,n} k^-_x h] ,$$

$$J_{m,n} [gh |\underline{k}^-|^2 - (\bar{\omega})^2] = - {}^3/_2 \frac{\bar{\omega} g}{h} a_{n2} a_{m1}$$

2.4 Expressions for a short crested sea

The expressions derived so far relate to just a pair of primary wave components. To define the long period disturbances associated with a short crested sea it is necessary to sum over all the pairs of wave components with amplitudes defined by the wave spectrum as given by (9) and (10). The resulting expressions for flat bed set-down have already been described (Ref.14). Here we consider the corrections to set-down as defined by (16) and a constant seabed slope of α as defined by (2).

For the measurements considered in the next section the directional spread in the wave spectrum was unknown and had to be inferred (Ref.14 explains the technique). We therefore assume the wave spectrum takes the form,

$$S_d (f,\Theta) = \frac{1}{(\pi\Theta_o)} \tfrac{1}{2} \exp (- \frac{\Theta^2}{\Theta_o^2}) \, S (f) \tag{17}$$

In this case, the familiar one dimensional spectrum S(f) is given by,

$$S\ (f) = \int S_d(f,\Theta)d\Theta\ ,$$

and Θ_o is a mean spread parameter for the wave spectrum.

We are now in a position to determine the one dimensional spectrum $S_c(f^-)$ which forms the spectrum of the corrections to the flat bed set-down spectrum due to varying water depth. The spectrum for these corrections takes full account of the directional spread of wave energy in the short crested sea: integrations over the angular spread can be performed to obtain the one dimensional spectrum. After considerable algebra, we find use of (16) yields

$$S_c(f^-) = \int S(f)\ S\ (f + f^-)\ I\ df \tag{18}$$

where,

$$I = \frac{0.0556 g^5 \tan^2\alpha}{\pi^9 h^7 \Theta_o} \left(\frac{h\pi}{2g}\right)^{1/2} \frac{1}{f^- f^4 (f+f^-)^4}\ ,$$

$$2\pi\ f^- = \omega^-$$

The spectrum $S_c(f^-)$ can be further integrated over difference frequencies and for typical parameters we then find the ratio of the pressure correction, due to varying water depth, over the pressure due to set-down is described by,

$$R = \frac{26.4\ T_p^{\ 2} \tan\alpha}{h^{3/2}}\ , \tag{19}$$

where T_p is the spectral peak period for the original wave spectrum and the constant has dimensions of $m^{3/2}\ s^{-2}$.

As this ratio approaches unity the assumption that corrections to the flat bed can be defined simply by expressions like (16) and (18) breaks down. We see the ratio increases dramatically as the water depth decreases. However, we also know the Stokes' expansion breaks down in shallow water which, in turn, invalidates all the solutions obtained here. For a wave of height H the limit of validity of the Stokes expansion is expected to be,

$$\frac{\lambda^2 H}{h^3} \leqslant 26$$

In the shallow water limit,

$$\lambda = \sqrt{gh}\, T,$$

and we find a limiting depth for validity of the theory given by,

$$h = 0.61\, H^{1/2} T_p \qquad (20)$$

Substituting this depth into the above ratio (19) gives,

$$R = \frac{55 T_p^{1/2}}{H^{3/4}} \tan \alpha$$

Taking the wave height H to be the significant wave height we find that bed slopes have to be steeper than about 1 in 100 for this ratio to approach unity for typical sea parameters where the long wave disturbances are likely to be important, e.g. significant wave heights in excess of 2m for spectral peak periods in the range 8s to 15s. Bed slopes are generally flatter than 1 in 100 for the depths of interest so, within the limits of validity of the Stokes' expansion underlying these solutions, the expressions (16) and (18) provide a reasonable estimate of the correction to the usual expressions for set-down.

We are now in a position to judge the validity of the solutions obtained so far, for real sea measurements of set-down. This aspect is considered in the next section.

3 REAL SEA MEASUREMENTS

It has already been mentioned that a method of analysis of real sea measurements, of pressure near the bed, has been developed to identify long period energy. This technique was applied initially to pressure sensor data collected off the outer end of the navigation channel leading to Port Talbot (Ref.14). Since that time, similar analyses have been performed for sites off Dover Harbour (Ref 17), Shoreham Harbour (Ref 18), Sunderland Harbour (Ref 19) and Barrow-in-Furness (Ref 20). The development of the analysis technique, together with its first application to the Port Talbot site, was funded under an earlier DOE strategic research contract. The subsequent applications to the 4 different harbour sites were carried out as project work for specific developments.

It is of interest to look for similarities in the 5 sets of data from the various sites as well as checking the validity of the analysis method in the light of the theoretical model, developed in Section 2, for set-down beneath wave groups on a sloping seabed.

3.1 Validity of analysis

The method of analysis already applied to site data makes use of the flat bed theory for set-down (Ref 14). Here we consider the magnitude of the corrections to this theory due to the sloping seabeds at the various sites.

At all the sites, the pressure sensor was mounted on the bed in a weighted frame as shown in Fig.2. The diaphragm of the sensor points upwards and is normally at a height of about 0.5m above the bed. The recorder used is the DNW-5 wave and tide recorder manufactured by NBA (Controls) Ltd. This instrument measures pressure by means of a precision piezoresistive transducer. The analogue pressure signals are converted into digitial form, in terms of centimetres of water, and recorded on a magnetic tape cassette at a frequency of 2 Hertz. Divers are used to retrieve the recorder for routine tape and battery changes every 6 weeks or so during a winter period. Recordings 2 hrs long are taken only when the significant wave height exceeds a pre-set threshold. Samples are taken regularly at certain times of the day to check the significant wave height. Exceedance of the threshold value then triggers a 2hr record. Such long records are needed when analysing data for long waves simply to reduce statistical uncertainty. For example, the 10 minutes of ordinary wave record, normally thought sufficient to define wave parameters, will contain 60 waves of 10s period. A 2hr record is needed to contain the equivalent number of waves of 2 minute period The other feature about the long period disturbance of interest is that they are correlated with the primary waves with set-down, in particular, expected to scale with the square of wave height. To ensure that such disturbances reach measureable proportions it is necessary, therefore, to restrict recording to those times when larger waves occur. Hence, the threshold value of significant wave height for 2hr records.

The analysis technique itself separates the total long period disturbance into set-down and surf beat. The two components are assumed to be uncorrelated with one another. The correlation measured between the set-down part of the long period disturbance and a wave envelope function enables the average directional spread of the ordinary waves (Θ_0 in equation (17)) to be inferred. This leads to an estimate for the set-down for each record using the flat bed theory (see equation (14) for the set-down potential for a pair of wave frequencies).

Now we have to allow for an additional long period potential of the type defined in (15) to be consistent with a sloping seabed. The presence of the additional velocity potential, however, still allows the flat bed set-down to be determined in the manner described above. Thus, having gone through the first stage we can then evaluate the disturbance associated with the additional potential. This process allows the relative magnitude of the correction to flat bed set-down to be estimated for each of the 5 measuring sites.

Port Talbot

Dealing first with Port Talbot, we consider two out of the ten 2hr records analysed previously. This data was collected at the offshore wave recorder site shown in Figure 3. The slope of the seabed is roughly 1 in 600, i.e. very small. The water depth at the recorder varied significantly due to the large tidal range. The pressure sensor is able to measure the depth for each record. We consider a typical record collected at the site. The primary significant wave height, spectral peak period and water depth were:

$H_s = 1.0\text{m}$, $T_p = 12.8\text{s}$, $h = 15.2$ m

Exact flat bed analysis of the record yielded the following magnitudes for set-down and surf beat (Ref 14).

H_s (set-down) = 0.022m, H_s(surf beat) = 0.047m.

A similar flat bed analysis but using shallow water wave theory yielded,

H_s(set-down) = 0.021m, H_s(surf-beat) = 0.048m.

It can be seen that shallow water theory leads to a good description of the long period disturbances and so we use (18) to evaluate the magnitude of the corrections to flat bed set-down. We call this correction "gradient forced,"

H_s(gradient forced) = 0.003m

As surf beat is defined here by the energy remaining after subtracting the energy in forced disturbances from the total we find, due to its small size, the presence of the extra gradient forced disturbance reduces the estimates of surf beat height by less than 1%. We can obtain this sort of result directly from (19). For the wave record in question R is 0.12 and taking 12% of set-down gives,

H_s(gradient forced) = 0.003m

The form of R shows that the largest gradient forced disturbance will tend to arise from relatively long period waves in shallow water. We therefore consider another 2 hr record with the same spectral peak period but a reduced water depth.

$H_s = 1.16\text{m}, \quad T_p = 12.8\text{s}, \quad h = 10.2\text{m}$

Exact flat bed analysis yields,

$H_s(\text{set-down}) = 0.073\text{m}, \; H_s(\text{surf beat}) = 0.074\text{m}$

Shallow water theory on a flat bed yields,

$H_s(\text{set-down}) = 0.072, \quad H_s(\text{surf beat}) = 0.075\text{m}$

Expression (18) leads to,

$H_s(\text{gradient forced}) = 0.018\text{m}$

The presence of this extra, gradient forced, disturbance only reduces estimates of surf beat height by 3%. A similar result can be obtained directly by calculating R from (19). We find R is 0.22 and 22% of set-down is 0.016m, leading to a reduction in estimated surf beat height of 2%.

These results show that use of the ratio R is sufficiently accurate to judge the importance of seabed slopes on the flat bed estimates of set-down and surf beat. The results are summarised in Table 1 for all the 2hr records from Port Talbot. We see that the gradient forced disturbance is negligible except for the first record where it reaches 22% of the set-down height, due mainly to a lower depth of water. But even for the first record, the estimate of surf beat height is only reduced by a few percent due to the presence of the forced disturbance. Thus, the conclusion for the Port Talbot data is that long period disturbances associated with the seabed gradient can be ignored, ie that flat bed theory can be used to calculate set-down and estimate surf beat height.

Dover Harbour

This long wave data was collected just south of the Admiralty Pier which forms the entrance to the Western Docks at Dover (Fig 4). The seabed slope leading up the the recorder is steeper than at Port Talbot, being in the region of 1 in 100. Examination of values of R (defined by (19)) for the 2hr records collected at Dover shows the following record to have the highest value.

$H_s = 1.88\text{m}$, $T_p = 9.3\text{s}$, $h = 17\text{m}$,
$H_s(\text{total long wave}) = 0.063\text{m}$

Flat bed analysis of the long period disturbances yielded,

$H_s(\text{set-down}) = 0.018\text{m}$, $H_s(\text{surf beat}) = 0.060\text{m}$

The value of R for this record is 0.33 and 33% of set-down gives,

$H_s(\text{gradient forced}) = 0.006\text{m}$

This in turn leads to a reduction in the estimate of surf beat height of just 0.5%

Analysis of other 2hr records shows a similar picture. Therefore, even though the seabed slope is much steeper than at Port Talbot there is only a small effect of the seabed gradient on the flat bed calculation of set-down and the estimation of surf beat.

Shoreham Harbour

In this case the pressure sensor was deployed south west of the harbour entrance (Fig 5). The seabed in the vicinity of the recorder is extremely flat, with the slope being approximately 1 in 700. The largest value of R for the 2hr records collected off Shoreham is associated with swell which must have originated in the Atlantic Ocean.

$H_s = 0.60m$, $T_p = 15.5s$, $h = 8.8m$,
H_s(total long wave) = 0.055m

Flat bed analysis of the long period disturbances yielded,

H_s (set-down) = 0.024m H_s (surf beat) = 0.049m

The value of R for the record is 0.35 and 35% of set-down is 0.008m, which leads to a reduction in the estimate of surf beat height of just 1.5%. The other 2hr records yield a similar picture thereby validating the flat bed calculation of set-down and estimation of surf beat.

Port of Sunderland

The location of the pressure sensor off Sunderland Harbour is shown in Figure 6. The seabed slope leading up to the recorder can be taken to be about 1 in 160. We find the largest value of R occurs for the following record,

$H_s = 2.59m$, $T_p = 12.5s$, $h = 17.4m$,
H_s(total long wave) = 0.136m

This condition appears to be associated with swell propagating down from the Northern North Sea. Flat bed analysis of the long period disturbance yielded,

H_s (set-down) = 0.097m, H_s(surf beat) = 0.095m

The value of R for the record is 0.36 and 36% of set-down is 0.035m but this reduces the estimate of surf beat height by only 7%. The other 2hr records are affected even less implying that, once again, flat bed theory can be used to separate out set-down and surf beat in the long wave records.

Barrow-in-Furness

Here, the pressure sensor was deployed south west of the Isle of Walney on the northern side of the outer navigation channel leading to Barrow-in-Furness. This position is indicated as 4 in Figure 7 where the channel can be identified by the finer refraction grid set-up to study the site. The slope of the seabed here is quite small, being about 1 in 600. We find the largest value of R occurs for the following record,

H_s = 1.65m, T_p = 8.4s, h = 5.7m,
H_s(total long wave) = 0.139m.

Flat bed analysis of the long period disturbances yielded,

H_s(set-down) = 0.102m, H_s(surf beat) = 0.094m.

The value of R for this record is 0.23 and 23% of set-down is 0.023m which leads to a reduction in the estimate of surf beat height of 3%. As the effect of the seabed gradient on the other 2hr records is even less, we are again justified in using flat bed theory to calculate set-down and estimate surf beat.

3.2 Surf Beats

The above results show that surf beat estimates obtained for the 5 sites in question after using the flat bed theory for set-down, hold good even though seabed slopes are as steep as in 1 in 100 in the vicinity of the pressure sensor. We are now in a position to look for similarities in the data from the different sites.

The purpose of the analysis carried out at Dover, Shoreham, Sunderland and Barrow was to extrapolate the measured long wave data to cover more extreme sea conditions than those measured. It has already been noted that surf beat height appears to scale more linearly with the ordinary wave height (Ref 9). This contrasts with set-down which scales with the square of wave height: hence the need to separate set-down and surf beat in the measured data before extrapolation. As set-down can be calculated for any given set of sea conditions (including extreme ones) using the validated flat bed theory, it is only necessary to extrapolate surf beat height to estimate the magnitude of the total long period energy in extreme seas. We therefore look for relationships between the significant height of surf beat and basic sea state parameters: significant wave height, spectral peak period and water depth, ie.

$$H_s(\text{surf beat}) \propto H_s^{\beta} \, T_p^{\gamma} \, h^{\delta}$$

This assumes, of course, that surf beats vanish in the absence of any waves (when $H_s = o$). The powers of the three parameters are chosen to minimise scatter in the data and we judge the scatter by a nomalised error parameter ϵ. The results obtained for the five sites are as follows:

	H_s (Surf beat)	ϵ
Port Talbot :	$0.00638 \dfrac{H_s^{1.32} T_p^{1.17}}{h^{0.34}},$	0.00055.
Dover :	$0.171 \dfrac{H_s^{0.96} T_p^{1.25}}{h^{1.55}},$	0.0012
Shoreham :	$0.00741 \dfrac{H_s^{0.93} T_p^{0.99}}{h^{0.06}},$	0.0031
Sunderland :	$0.00106 H_s^{0.86} T_p^{1.61} h^{1.55},$	0.0056
Barrow :	$0.00264 \dfrac{H_s^{1.08} T_p^{1.59}}{h^{0.036}},$	0.0040

Depth variation for the Dover and Sunderland sites is clearly very different from that at the other three sites. Implicit in the approach adopted so far is that scatter in the data has been reduced by subtracting off the calculated set-down energy for each record. We can check this by looking for similar fits to the total measured long wave height

	H_s (total long wave)	ϵ
Port Talbot :	$0.0217 \dfrac{H_s^{1.49} T_p^{1.27}}{h^{0.84}},$	0.00067.
Dover :	$0.168 \dfrac{H_s^{1.08} T_p^{1.27}}{h^{1.55}},$	0.0011

Shoreham : $0.107 \dfrac{H_s^{1.09} T_p^{1.08}}{h^{0.26}}$, 0.0032

Sunderland : $0.00339 H_s^{0.96} T_p^{1.78} h^{1.03}$, 0.0049

Barrow : $0.00468 \dfrac{H_s^{1.16} T_p^{1.92}}{h^{0.80}}$, 0.0097

The first point to notice is that, except for Barrow, the scatter in the data is not very different for the two sets of results. This is explained by set-down generally being noticeably smaller than surf beat except at Barrow where set-down sometimes exceeded surf beat. Also, comparison of the above set of results with the set for surf beat height does show a reduction in scatter for Port Talbot, Shoreham and Barrow when set-down energy is taken out of the equation, but the scatter increases for Dover and Sunderland. This indicates that the analysis is not working as expected for the latter two sites. One reason could be associated with reflections off the breakwaters. It was clear in the physical model investigations subsequently carried out at HR for the Dover and Sunderland Harbour Boards, that reflections from the vertically faced breakwaters radiated noticeable amounts of energy back out to sea, Figures 4 and 6 show that, depending on wave direction, the pressure sensor could have experienced some of this energy although the height of the reflection must fall off with distance from the breakwater as energy spreads out offshore. The likely effect on the analysis of any primary wave reflections would be to cause an overestimate of the set-down. Then, subtracting off too much set-down energy could increase, instead of decrease, the scatter. In these cases it may be better to obtain a fit to the total long wave height using the records for which calculated set-down (possibly overestimated) is small relative to the total long wave height, ie using the records that are dominated by surf beat. It might be thought the data from Port Talbot would also contain primary wave reflections from the harbour breakwaters (Fig 3) but in this case the breakwaters are of rubble mound construction and expected to produce minimal primary reflection effects at the site of the wave recorder. Considering Port Talbot, Shoreham and Barrow together we can average the powers of wave height, period and depth for the 3 sites to give,

$$H_s(\text{surf beat}) \propto \frac{H_s^{1.11} T_p^{1.25}}{h^{0.25}} \qquad (21)$$

This relationship is consistent with the near linear dependence of surf beat on wave height found by Tucker (Ref 9). The inverse quarter power of depth dependence is consistent with the inverse shoaling of a long wave expected as surf beats propagate out to sea.

It is then of interest to plot the data for these 3 sites assuming the dependence shown in (21). These plots appear in Figures 8, 9 and 10.

In the case of Port Talbot (Fig 8) 4 records collected at an earlier date, that were only ½ hour long, are also shown because they relate to more extreme conditions where the significant wave height reached 4m in some cases. The resulting surf beat estimates are also higher and appear in reasonable agreement with the best fit line for the 2hr records, all of which relate to smaller heights (Table 1). The larger scatter for the ½hr records is at least partly due to the greater statistical uncertainty arising from what are relatively short records for long waves.

Surf beats off Shoreham Harbour are shown in Figure 9. The best fit line is seen to be steeper than that for the Port Talbot data.

Surf beats off Barrow appear in Figure 10. Here there is considerably more data, some 90 records, than for the other two sites but the best fit line has a slope similar to that for the Port Talbot data.

Comparison of Figures 8, 9 and 10 shows an increasing scatter in the data which is consistent with an increasing value for ϵ, the normalised error parameter, tabulated above for surf beats. One possible explanation for the different amounts of scatter at the various sites is that different wave directions produce different amounts of surf beat. The wave direction for the data collected at Port Talbot can be expected to be fairly constant, ie from the south west, as the relatively long period of the waves in question (Table 1) shows they originate in the Atlantic. This may explain the small amount of scatter in the 2hr records. The larger scatter in the ½hr records may be statistical, as already mentioned. A greater range of wave directions will be present in the Shoreham data possibly increasing scatter, while such scatter is likely to be further enhanced at Barrow by the changing shoreline formed by the Isle of Walney (Fig 7).

There is insufficient data for Port Talbot and Shoreham to attempt a subdivision of the data according to wave direction but the relatively large amount of data from Barrow does allow consideration of wave direction. This was done by making use of results from HINDWAVE which hindcasts waves from wind data (Ref 21) and so provides estimates of wave direction as well as height and period. In fact the HINDWAVE model was used in the project work for Barrow and, in the process, the model was calibrated against measured wave height/period data (Ref 20)

The results of the subdivision of the Barrow surf beat data according to offshore (deep water) wave direction is shown in Figures 11 to 15. It can be seen the scatter in the data is reduced in Figure 11 (180 - 200°N) Figure 12 (200-220°N) and in Figure 15 (260-300°N). But significant scatter remains in the data for 220-260°N (Figs 13 and 14). It is possible that the "elbow" in the Isle of Walney causes greater scattering of the surf beats reaching the recorder for 220-260°N than for the other incident wave directions. This would also be consistent with a reduced surf beat height for 220-260°N as shown by the best fit lines in Figures 13 and 14. Thus, the subdivision of data according to wave direction does appear to reduce scatter in the data for some directions but the picture is complicated by the relatively complex shoreline of the Isle of Walney.

On the whole, the best fit lines through the data from the three sites (Figs 8, 9 and 10) show remarkably similar constants of proportionality considering the very different wave conditions involved ie.

$$H_s \text{ (surf beat)} = K \frac{H_s^{1.11} T_p^{1.25}}{h^{0.25}}$$

where K has the values 0.0044, 0.0066 and 0.0041 for Port Talbot, Shoreham and Barrow, respectively.

These results can be used to give some idea of the total long wave climate that can be expected at positions just offshore of harbour entrances at Shoreham and Barrow (there is insufficient data to do this for Port Talbot).

Considering Shoreham first, we allow for high water when wave heights are expected to be at their maximum due to reduced bottom friction effects. Thus, at the recorder position (Fig 5) the water depth is taken to be 12.4m. The mean directional spread parameter in the primary waves (needed to evaluate the set-down component of the total long wave energy) was taken to be average of the spread parameters estimated from representative measured data, ie 11.5°. Extreme conditions for the recorder position at Shoreham were estimated from wave recording and prediction exercises for all directions combined (Ref 18). The following primary and long wave parameters are then obtained for various return periods.

Return period	Primary waves		Significant long waves (m)		
	H_s(m)	T_p(s)	H_s(set-down)	H_s(surf beat)	H_s(total)
10 times/yr	3.3	7.5	0.14	0.16	0.21
1/yr	4.0	8.4	0.27	0.23	0.35
1/10yrs	4.7	9.2	0.45	0.31	0.55
1/100yrs	5.4	10.0	0.71	0.41	0.82

These results indicate that surf beat dominates set-down for frequently occuring conditions but set-down becomes dominant, due to its square law dependence on wave height, for more extreme sea states.

A similar exercise for Barrow gives the following results for the wave recorder position (Fig 7) at high water (13.1m depth) for the direction sector with the largest waves (240-260°N).

Return period	Primary waves		Significant long waves (m)		
	H_s(m)	T_p(s)	H_s(set-down)	H_s(surf beat)	H_s(total)
10 times/yr	2.4	7.5	0.10	0.071	0.12
1/yr	4.0	9.2	0.32	0.16	0.36
1/10yrs	4.8	10.0	0.53	0.22	0.57
1/100yrs	5.4	10.5	0.75	0.27	0.80

Here, even for the 10 times a year condition, set-down is larger than surf beat at the recorder position. However, within the shelter of the Isle of Walney it seems likely that the total long period disturbance will be the residue of surf beat rather than set-down. This is because refraction reduces the primary waves by the time they reach the eastern tip of the Isle of Walney implying that the associated incident set-down will be much reduced. Surf beat, on the other hand, can be expected to be similar in magnitude to the heights given above at this position.

These two sets of results indicate that the significant height of surf beat in intermediate water depths typical of harbour entrances can be expected to approach 0.3m to 0.4m in extreme conditions. They also indicate that surf beat is important in describing long waves in harbours, particularly for the more frequently occuring sea states.

4 POSSIBLE MECHANISMS FOR SURF BEAT GENERATION

The previous section has provided evidence that surf beat energy exists in the real sea in water depths typical of harbour entrances. The analysis technique allows for the forced disturbance to be subtracted from the total measured long period energy. The remaining energy is considered to be due to surf beats under the definition of surf beat adopted here. Typical magnitudes of surf beat heights in intermediate water depths (position of wave recorder) have been presented and shown to be important for harbour and moored ship response.

Many mechanisms have been proposed to explain the generation of surf beats. It is convenient to subdivide these into mechanisms for onshore/offshore motion and mechanisms for edge wave, or alongshore, motion. In onshore/offshore motion, standing waves are formed by reflection from the shoreline and, in simple cases, one expects nodes and antinodes to be lines parallel to the shoreline. This type of wave is sometimes called a "leaky" mode as it radiates energy in the seaward direction. This contrasts with edge waves which propagate parallel to the shoreline so that energy remains trapped in the nearshore region. In the case of simple standing edge waves, nodes and antinodes are lines perpendicular to the shoreline.

4.1 Onshore/offshore surf beats

This type of motion has already been mentioned in Section 1.2. There are two mechanisms for such motions in the infagravity range of frequencies.

One, proposed by Longuet-Higgins and Stewart (Ref 2) involves the release of the energy present in set-down beneath wave groups when the ordinary waves break on a beach. Within the surf zone, the wave grouping modulation is destroyed as waves tend to adjust their height in proportion to the local depth, losing energy through breaking as they propagate up the beach. It was suggested in Reference 2 that set-down energy, which is tied to groups of waves propagating toward the surf zone, is released at the breaker line as free long wave energy which reflects from the beach to produce seaward going surf beats. Such waves would explain Tucker's observations of surf beats with a negative correlation with groups of waves passing the recorder at an earlier time. This delay being consistent with the time taken for groups of waves to reach the surf zone and long waves to travel back past the recorder. However, no adequate theory describing the release of set-down energy in the vicinity of the surf zone has yet been produced. In addition, there is no adequate theory describing set-down just offshore of the surf zone. As already mentioned this is due to the breakdown of the Stokes' expansion of the wave equations used to describe set-down in deeper water.

Although we have found the Stokes' expansion works well in intermediate depths, breakdown occurs in shallower water when finite wave amplitudes begin to affect the propagation of ordinary waves (see (20) in Section 2.4). There is, however, some evidence of the release of set-down energy in the form of surf beats in flume studies carried out by Bowers (Ref 3) and Flick et al (Ref 22). But these studies suffered from problems of reflection of surf beats from the wave-maker plus, in the case of Reference 22, spurious long waves due to not compensating for set-down at the paddle. Such experimental difficulties make it impossible to verify the proposed mechanism. Very careful flume studies are needed to investigate the problem further and verify the proposed mechanism (see Recommendations).

The second mechanism for the generation of onshore/offshore surf beats is that proposed by

Symonds et al (Ref 10). This was mentioned in Section 1.2. Here, the oscillation of the break point up and down a beach slope, as waves of varying height within a group break in varying depths, produces both shoreward going and seaward going long waves. The latter plus the reflection of the former from the shoreline form the final seaward going surf beat. The equations used in Ref.10 are the following,

$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = \frac{1}{\rho h}\frac{\partial}{\partial x}S_{xx} \qquad (22)$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(hu) = 0 \qquad (23)$$

Here, u is the depth intergrated horizontal velocity, ρ is the density of water and S_{xx} is the radiation stress representing the onshore flux of momentum.

It is assumed that:

$$S_{xx} = \tfrac{3}{4}\rho g a^2 \qquad (24)$$

where a is the incident wave amplitude. Outside the surf zone the special case is taken of a sinusoidal variation in wave amplitude at the group frequency while inside the surf zone the amplitude is taken to be proportional to the depth,

$$a = \gamma\, x\tan\alpha, \; 0.4 \leqslant \gamma \leqslant 0.8 \qquad (25)$$

Any set-down outside the surf zone is neglected with the mean sea surface taken to be the still water level. The surface elevation is scaled relative to the steady state set-up obtained by integrating (22) from the shoreline to the mean position X of the break point. Using (24) and (25) to substitute for the forcing terms on the right-hand side of (22) it is possible to express the forcing term as a Fourier series containing the group frequency and its harmonics. In this way the solution to the equations is found to consist of long waves at the group frequency and its harmonics radiating both shoreward and seaward. It can be seen that the theory, as presented so far, requires extension to the case of a random sea before it can be verified in laboratory work using realistic seas. Some flume work carried out for the special case of just two wave frequencies, i.e regular wave groups, has produced qualitative but not quantitative agreement with this theory (Ref.6). But again, more extensive flume tests are needed to really verify this proposed mechanism of surf beat generation.

4.2 Edge Waves

Here we consider the mechanisms that have been proposed for the generation of alongshore motion, or edge waves, in the infragravity range of frequencies. There appear to be two basic mechanisms.

(i) If the beach reflects the primary waves then standing subharmonic edge waves at twice the wave period can grow through non-linear interactions. Thus, wave periods of 10 to 20 seconds can generate edge waves of period 20 to 40 seconds.

(ii) On dissipative beaches incident waves of certain frequencies and alongshore wave numbers can force an edge wave at a frequency given by the difference of the two wave frequencies.

Dealing first with subharmonic edge waves, Bowen and Inman (Ref 23) identified standing edge waves at periods similar to that of the waves. They found that subharmonic edge waves were commonly excited when conditions were such that waves were reflected from beaches. This differs from the situations where wave breaking dissipates most of the wave energy. In the latter case, the second mechanism described above is needed to generate low frequency edge waves. Following observations of subharmonic edge waves (Ref 23) theoretical work was carried out (Refs. 24,25,26) into the resonant formation of a standing edge wave of period 2T by normally incident and reflected waves of period T. This work made use of non-linear shallow water equations (Ref 25) and the general theory of water waves (Ref.26) to show how subharmonic edge waves are generated. Initial growth rates are exponential but on reaching a finite amplitude, non-linear terms limit the growth of the edge wave. There are two factors here. One, the finite amplitude edge wave radiates energy offshore at a period of half that of the edge wave. Two, finite amplitude effects alter the frequency of the edge wave via amplitude dispersion and so detune it from the energy source formed by the primary waves. This clearly limits the growth of the edge wave. Another important factor limiting the growth of edge waves comes about if the waves change the topography of the beach. Labaratory experiments with a mobile bed carried out by Guza and Inman (Ref 27) suggest that subharmonic edge waves produce beach cusps which grow in size until the edge waves linked to their formation decay away.

The subharmonic edge waves produced when the incident waves make an angle with the beach have also been studied and shown to be progressive instead of standing (Ref 28).

The edge waves generated by the above mechanism are still of too short a period to explain the complete low frequency edge wave spectrum observed in nature (Ref 29). However, it is well known that wave-wave interactions in the deep sea shape the spectrum of energy and it has been shown using shallow water equations on a beach of constant slope (Ref 30) that similar interactions can be expected between edge waves. Here such interactions could transfer energy from subharmonic frequencies to lower frequencies, i.e. to longer period edge waves.

The second basic mechanism for edge wave generation was proposed by Gallagher (Ref 31). In this case the edge waves, with periods covering the infravity range, are generated directly. He used the shallow water wave equations and made a number of assumptions. The theory applied to dissipative beaches where all the primary wave energy is lost through wave breaking. Wavelengths were assumed constant during shoaling (they are reduced according to linear theory) while wave amplitudes were taken to grow exponentially (they grow only weakly according to $h^{-1/4}$ in linear theory). The beach slope was assumed constant, linear bottom friction was assumed to limit the growth of edge waves and the incident wave spectrum was assumed to be narrow banded. Gallagher was then able to show that two wave components in the incident spectrum with frequencies ω_1, ω_2 and alongshore wave numbers k_{y1}, k_{y2} are able to force an edge wave at the difference frequency provided

$$(\omega_2-\omega_1)^2 = (k_{y2}-k_{y1})\ g\ (2n+1)\ \alpha \qquad (26)$$

Equation (26) is the usual edge wave dispersion relationship for mode n on a beach of constant slope α. In practice, only a finite number of modes are possible: the requirement being (Ref 32)

$$(2n+1)\ \alpha \leqslant {}^{\pi}/_{2}$$

Gallagher considered only the 3 lowest edge wave modes in integrating over the interactions within the shoaling wave spectrum so the applicability of his results to realistic sea conditions may be restricted.

However, Bowen and Guza (Ref 33) were able to confirm that Gallagher's mechanism of edge wave generation occured in the laboratory for simple cases consisting of two wave components.

5 CONCLUSIONS

1. Real sea measurements have shown that there are two components to infragravity waves, set-down and surf beat, and that both are important in harbour design. This is due to the sensitivity of moored ships to long period forcing: resonances can occur that cause berth downtime due to increased horizontal ship movements and mooring difficulties.

2. Past laboratory work has enabled a flat bed theory describing set-down in intermediate depths, typical of harbour entrances, to be verified. In this report the flat bed theory has been extended to allow for the more realistic case of a sloping seabed. This is in recognition of the fact that set-down will be more sensitive to seabed slope than the ordinary waves due to its longer wavelengths.

3. Applying the extended theory to long wave data collected by Hydraulics Research off the entrances to five harbours: Port Talbot, Dover, Shoreham, Sunderland and Barrow-in-Furness, it has been possible to show that corrections to the flat bed theory of set-down appear in the form of an additional disturbance forced by wave grouping. This disturbance increases as the water depth decreases and although it can approach a significant percentage of the set-down for some of the wave records at low tide: up to 22% at Port Talbot, 33% at Dover, 35% at Shoreham, 36% at

Sunderland, 23% at Barrow, it has been demonstrated that the flat bed theory for set-down is sufficiently accurate when it comes to estimating surf beat energy in the measured data. This happens because the additional forced disturbance is not in phase with set-down and because in energy terms, it formed a small percentage of the total long period energy measured at the five sites.

4. The above finding concerning the accuracy of using a flat bed theory of set-down for realistic seabed slopes is of importance in modelling waves in harbour studies. It shows that in water depths typical of those represented at the wave-maker in physical models (normally greater than the depths at the wave recorders mentioned above) it is only necessary to compensate for set-down beneath wave groups: the energy in the additional forced disturbance associated with the local seabed gradient being too small to matter. Thus, the techniques of set-down compensation already developed at Hydraulics Research specifically for physical models of harbours (Ref 7) do not require modification in this regard.

5. Further examination of the surf beat energy measured at the five harbour sites shows that it scales roughly linearly with wave height and that it can be expected to reach significant heights of 0.3 to 0.4 m in severe storms. Such levels of long wave activity are known to be capable of closing down a harbour (Ref 1) due to mooring difficulties. Of course, set-down will also be contributing to the total long period energy incident on harbours and in very servere storms it can be expected to be the dominant component because it scales with the square of

wave height instead of the more linear scaling of surf beat. However, in more frequently occuring storms the measurements indicate that surf beat can exceed set-down. Thus, both set-down and surf beat are important components of the total long period disturbance present at harbour entrances.

6. Many mechanisms have been proposed in the literature for the generation of surf beats and some laboratory tests of idealised situations have provided qualitative agreement for some of the theories. But the situation remains confused and it is unclear from available measurements which mechanisms are the important ones in the real sea. This is because the ordinary waves are highly non-linear in shallow water where surf beats are thought to be generated. The directional spread of wave energy and the effects of the tide also complicate picture for real sea data. This means no adequate theory exists to explain the surf beats measured at the five harbour sites mentioned above. Thus, more research is needed to ensure that surf beats are well represented in harbour modelling.

7. The levels of surf beat activity measured at some distance from the coastline near the five harbours (Figs 3 to 7) indicate that an important component of the total surf beat spectrum will be associated with long waves propagating seawards. This is because their heights appear to scale with water depth to the minus quarter power ie the inverse of a shoaling wave. It shows that surf beats will not consist entirely of edge waves which remain trapped by the coastline with heights that decay more rapidly with distance from the shorelines. This has important consequences for harbour modelling. The techniques being developed at

Hydraulics Research under the existing DOE contract will enable more realistic short crested waves to be used in harbour modelling. But wave guides are necessary to maintain the wave height up to the harbour (Ref 13) which in turn means any surf beat returning from the model coastline will become trapped between the guides and tend to re-reflect from the wave-maker. This seems certain to lead to a poor representation of surf beats in the physical model. The problem can, however, be addressed by absorbing any returning long waves at the wave-maker. This only becomes possible because a short crested wave-maker has multi-elements enabling waves returning at an angle to be absorbed provided the appropriate signal generation system can be developed.

6 RECOMMENDATIONS

From the discussion presented in this report it is apparent that, although important for harbour design, surf beats are poorly understood. There is no shortage of mechanisms to explain their generation but the complexity of real sea data has prevented verification of these mechanisms. Also, in most cases, the theory for these mechanisms has not been generalised to the realistic case of a wave spectrum. The obvious way forward in this situation is to carry out careful laboratory experiments to identify the important mechanisms causing surf beats and to provide data for validation of theoretical work. Such experiments will need to allow for compensation for set-down and absorption of surf beats at the wave-maker to avoid spurious long waves contaminating the experiments. Tests are needed initially in a flume for onshore/offshore surf beats and then in a wave basin using short created waves for edge wave type surf beats. In developing the necessary signal generation system for these experiments the problem of unrealistic re-reflection of surf beats by a short crested wave-maker in physical harbour models will have been solved. This in turn will improve the accuracy of the physical model which over the years has proved itself to be the most accurate type of model for harbour design.

7 ACKNOWLEDGEMENTS

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TABLES.

TABLE 1 LONG PERIOD DISTURBANCES OFF PORT TALBOT (Fig 3)

Water depth	Primary waves		Measured long waves	Flat bed theory		Gradient forced	
(m)	$\underline{H_s}$(m)	$\underline{T_p}$(s)	$\underline{H_s}$(m)	$\underline{H_s}$(set-down)	$\underline{H_s}$(surf beat)	$\underline{H_s}$(m)	Reduction in surf beat height
10.2	1.16	12.8	0.104	0.073	0.074	0.016	2%
13.5	1.72	11.1	0.100	0.058	0.081	0.006	0.5%
13.8	0.88	12.8	0.052	0.029	0.043	0.004	0.5%
14.9	2.00	9.3	0.111	0.074	0.083	0.005	0.2%
15.2	1.00	12.8	0.052	0.022	0.047	0.003	0.2%
15.8	1.24	12.8	0.070	0.036	0.060	0.004	0.2%
16.2	0.88	12.8	0.042	0.012	0.040	0.001	0.1%
18.4	1.12	15.2	0.074	0.024	0.070	0.003	0.1%
18.9	1.52	12.8	0.092	0.031	0.087	0.003	0.05%

FIGURES

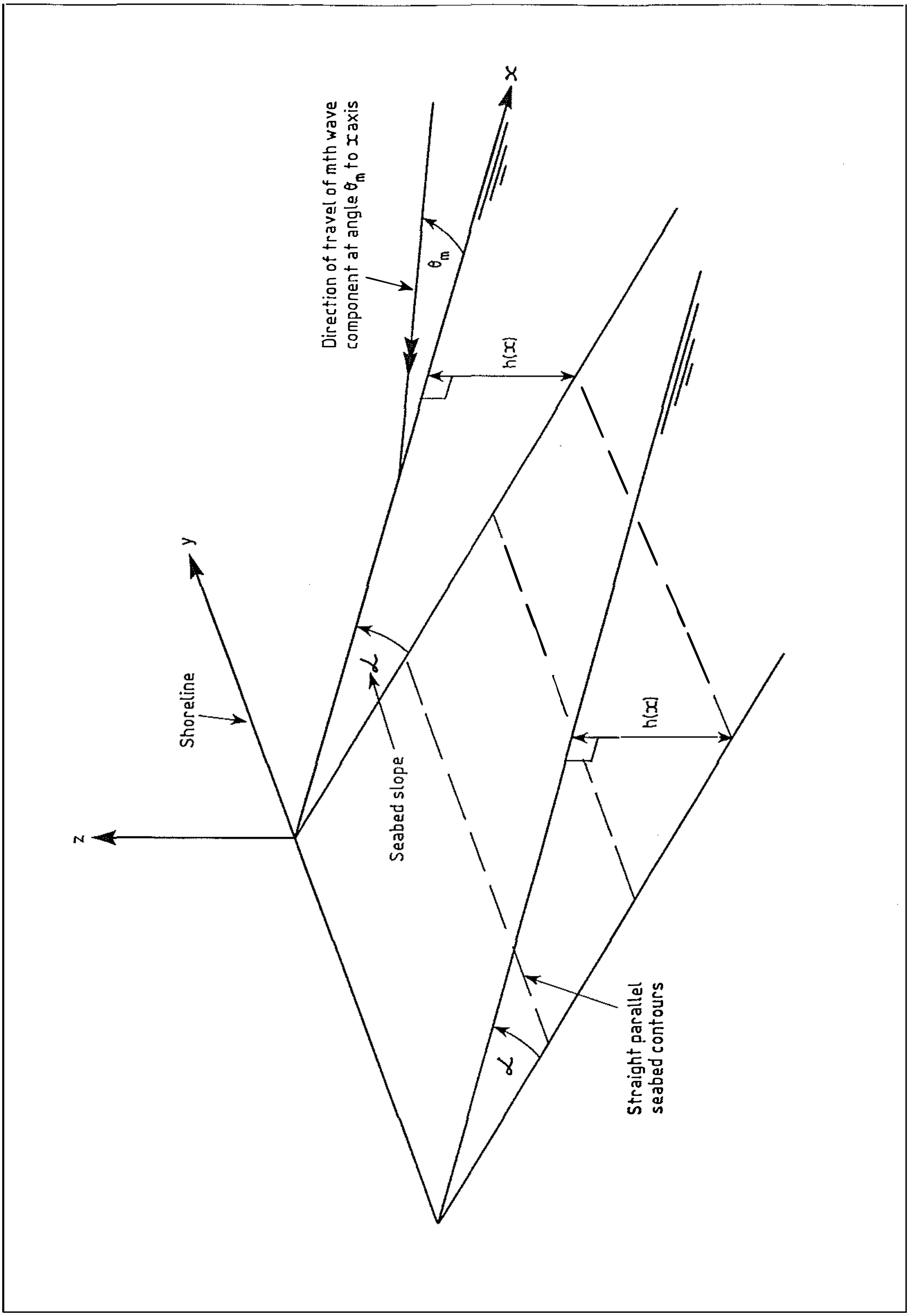


Fig 1 Definition of coordinates and seabed configuration

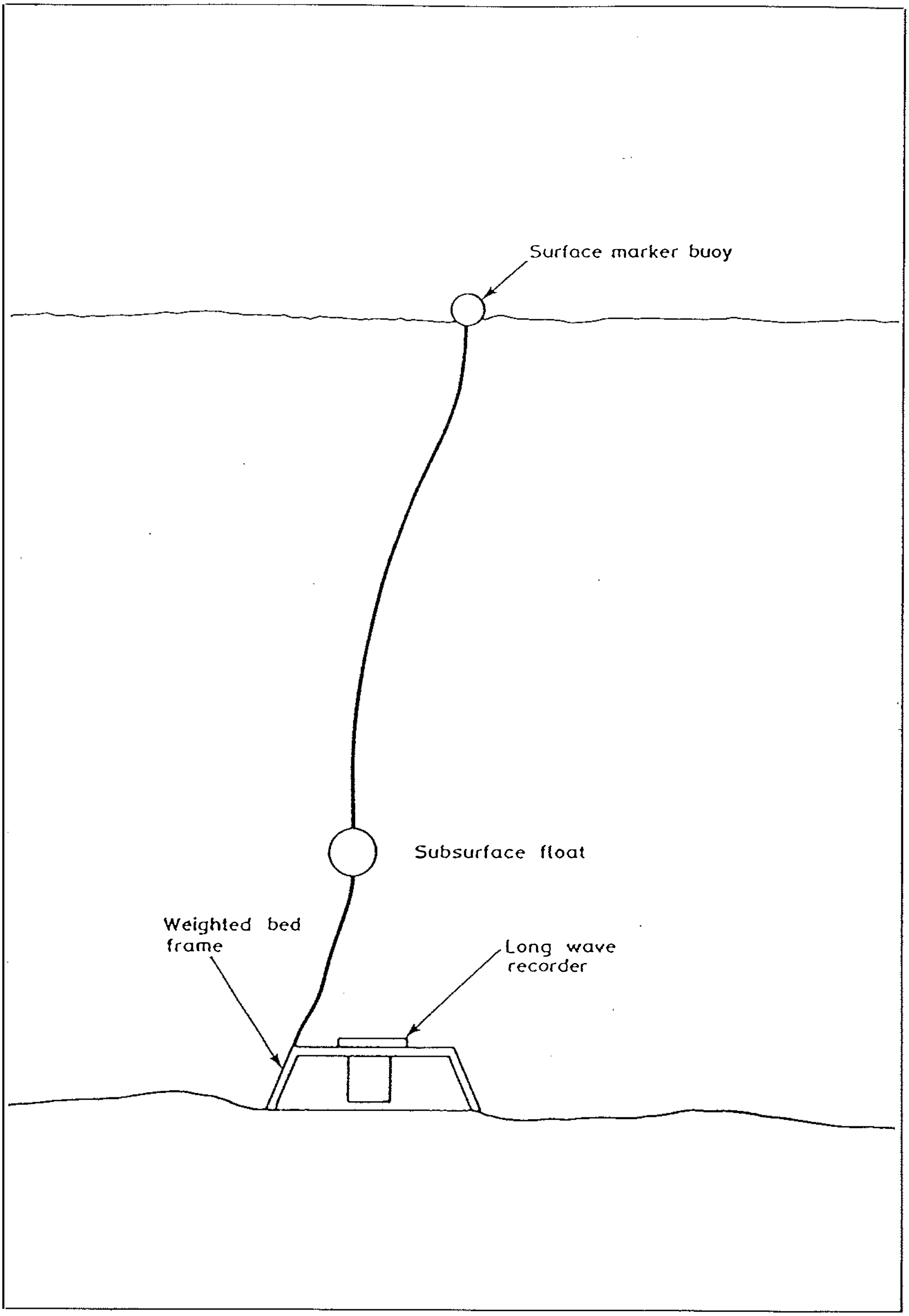


Fig 2 Wave recorder mooring arrangement

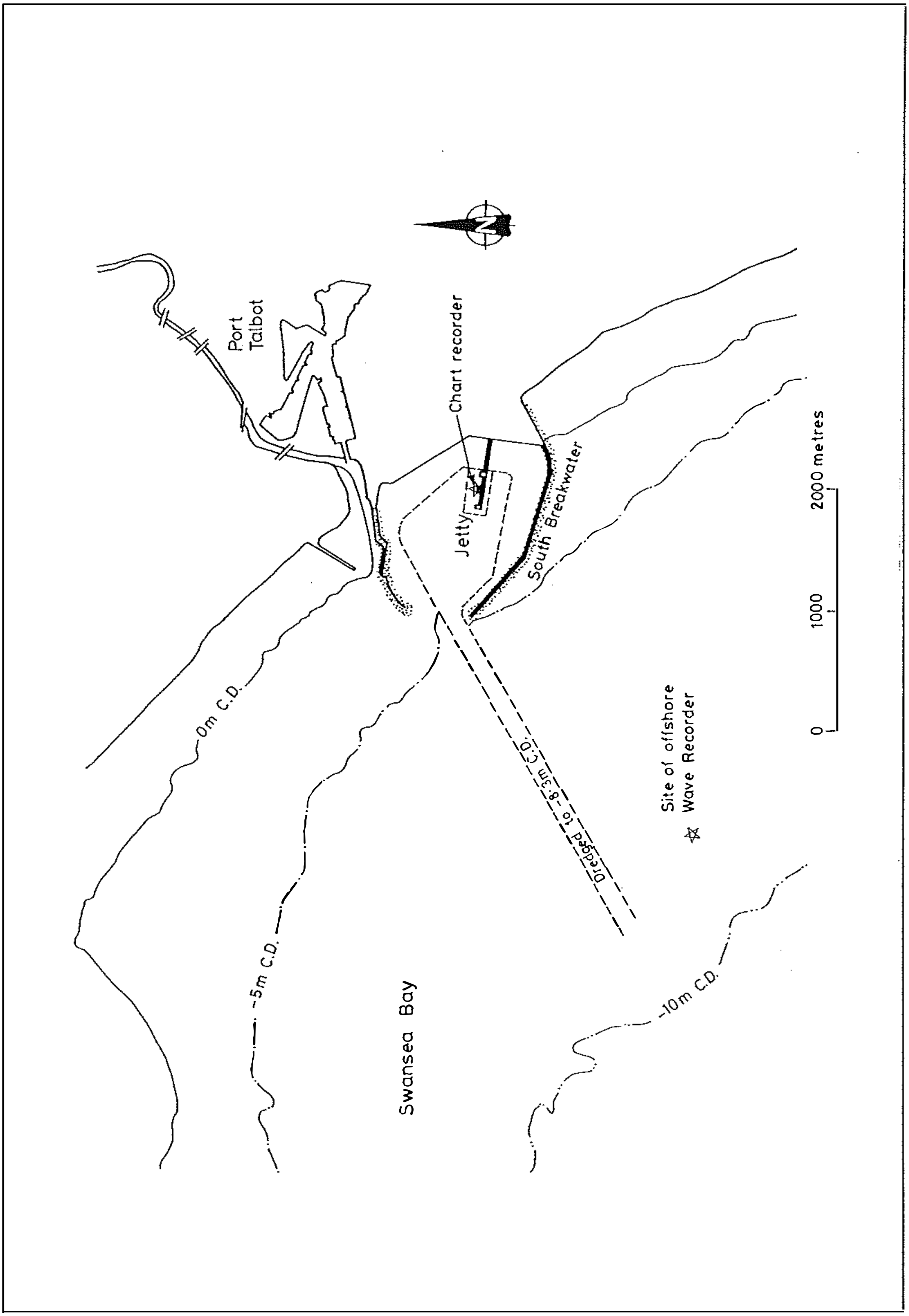


Fig 3 Port Talbot - wave recorder position

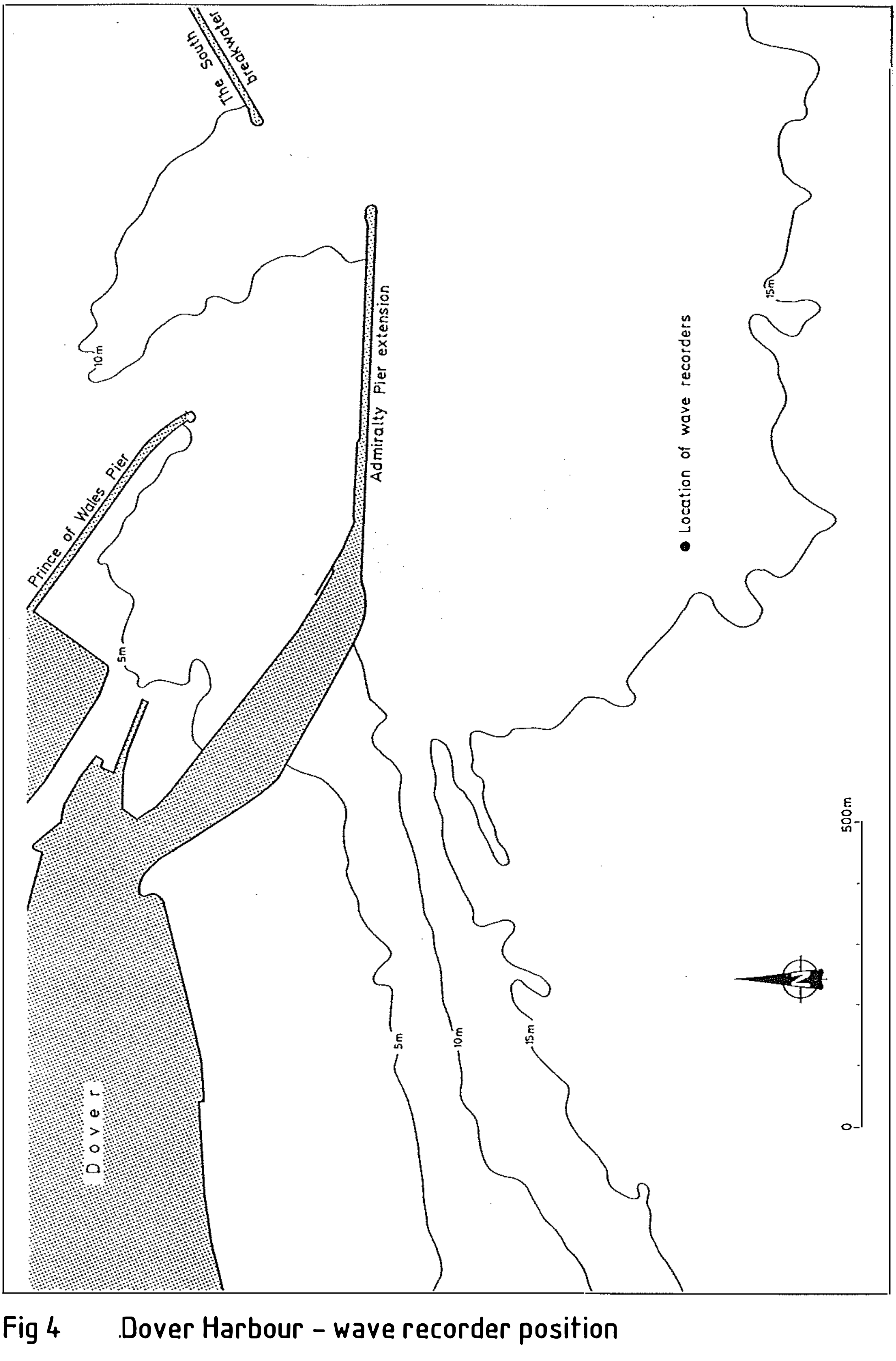


Fig 4 Dover Harbour - wave recorder position

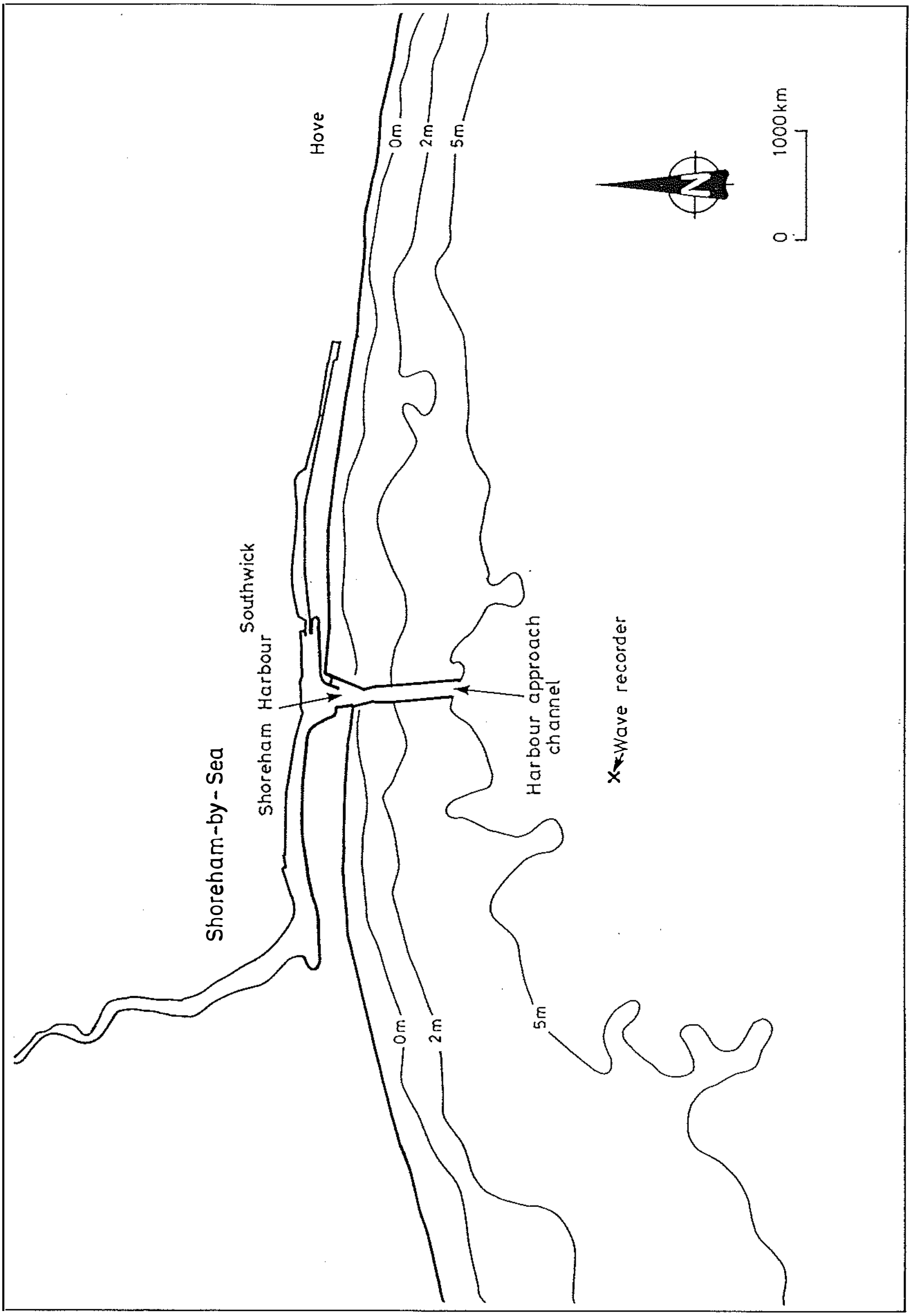


Fig 5 Shoreham Harbour - wave recorder position

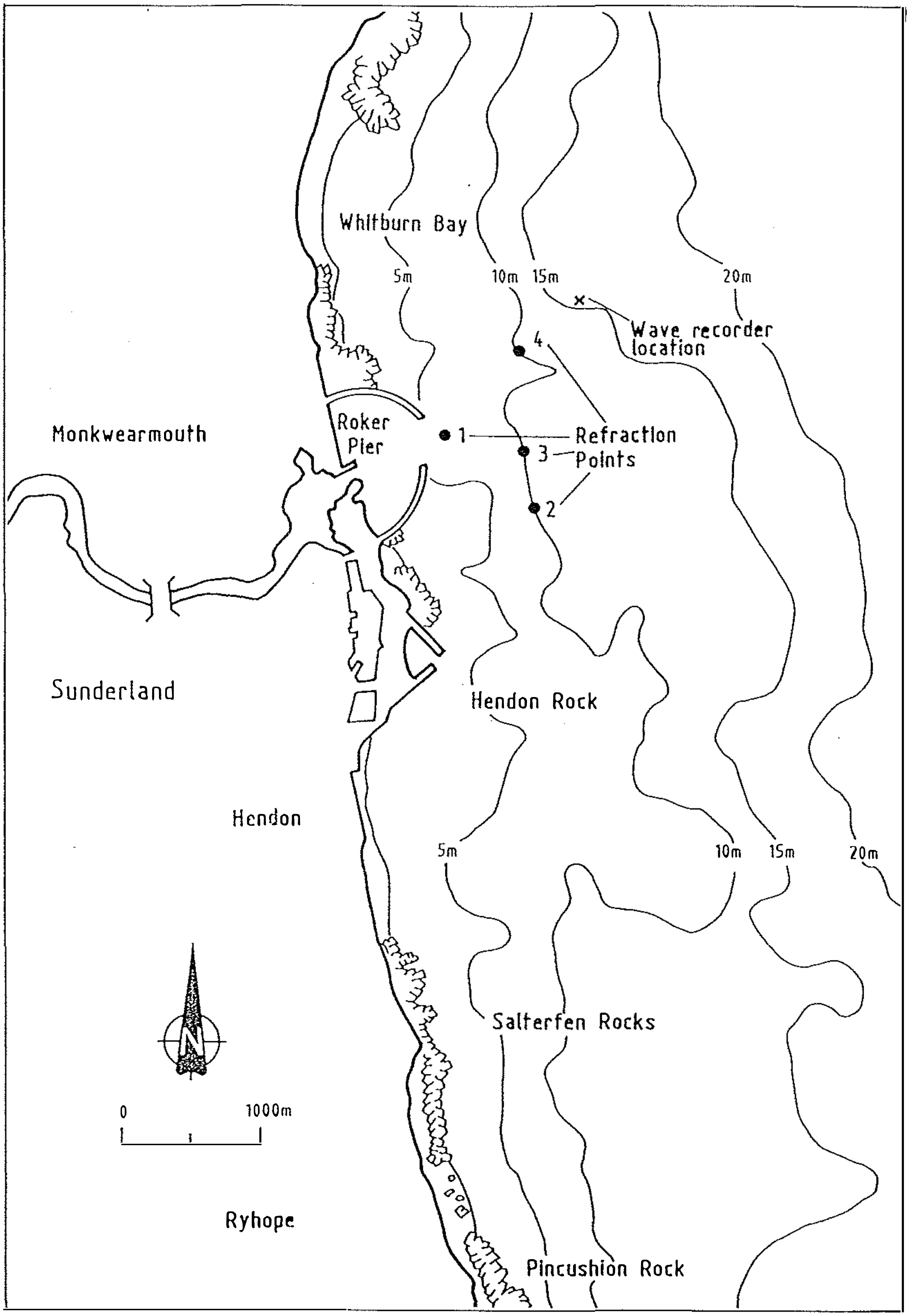


Fig 6 Sunderland Harbour - wave recorder position

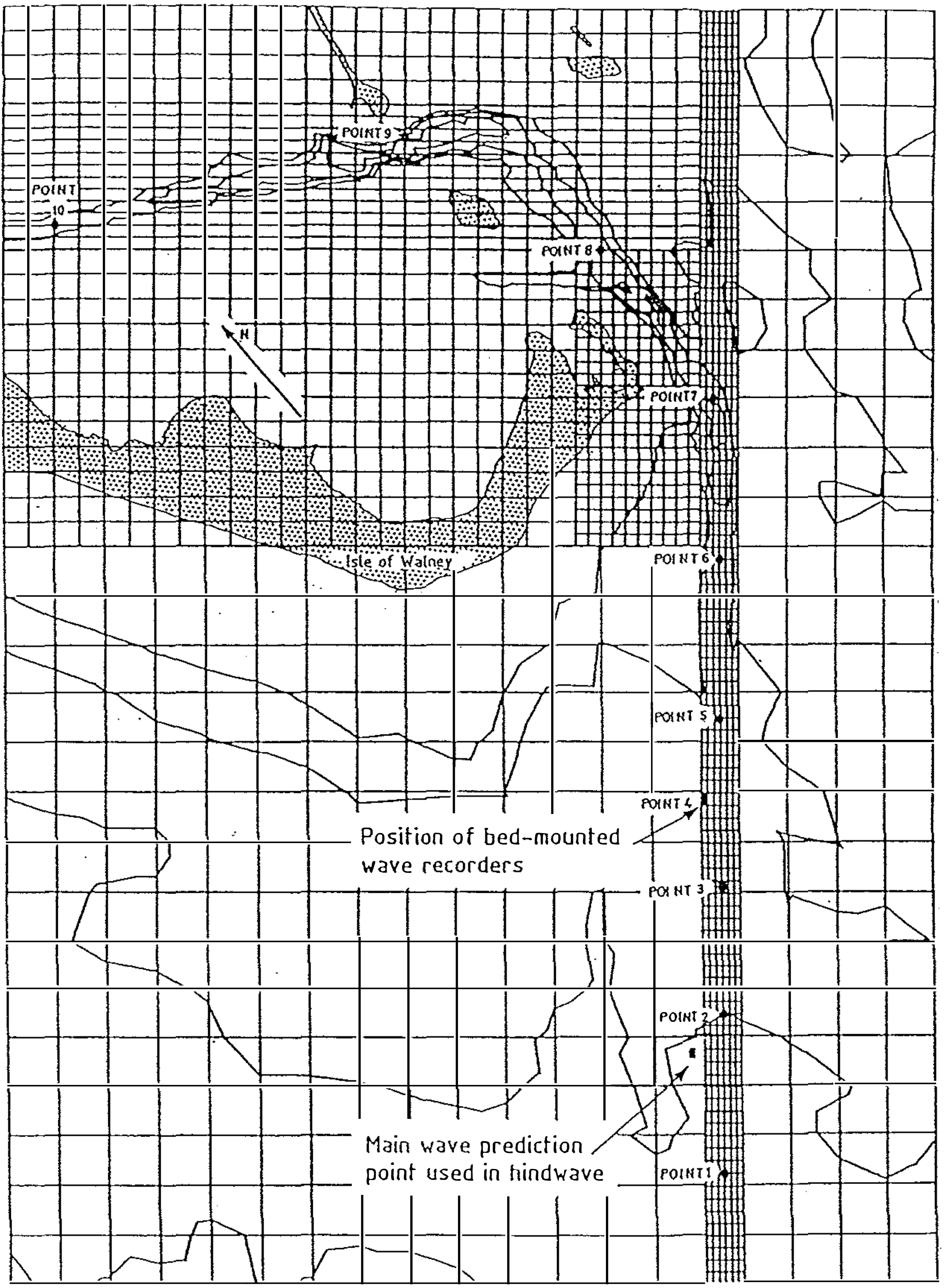


Fig 7 Barrow-in-Furness Harbour - wave recorder position

Fig 8 Surf beats off Port Talbot

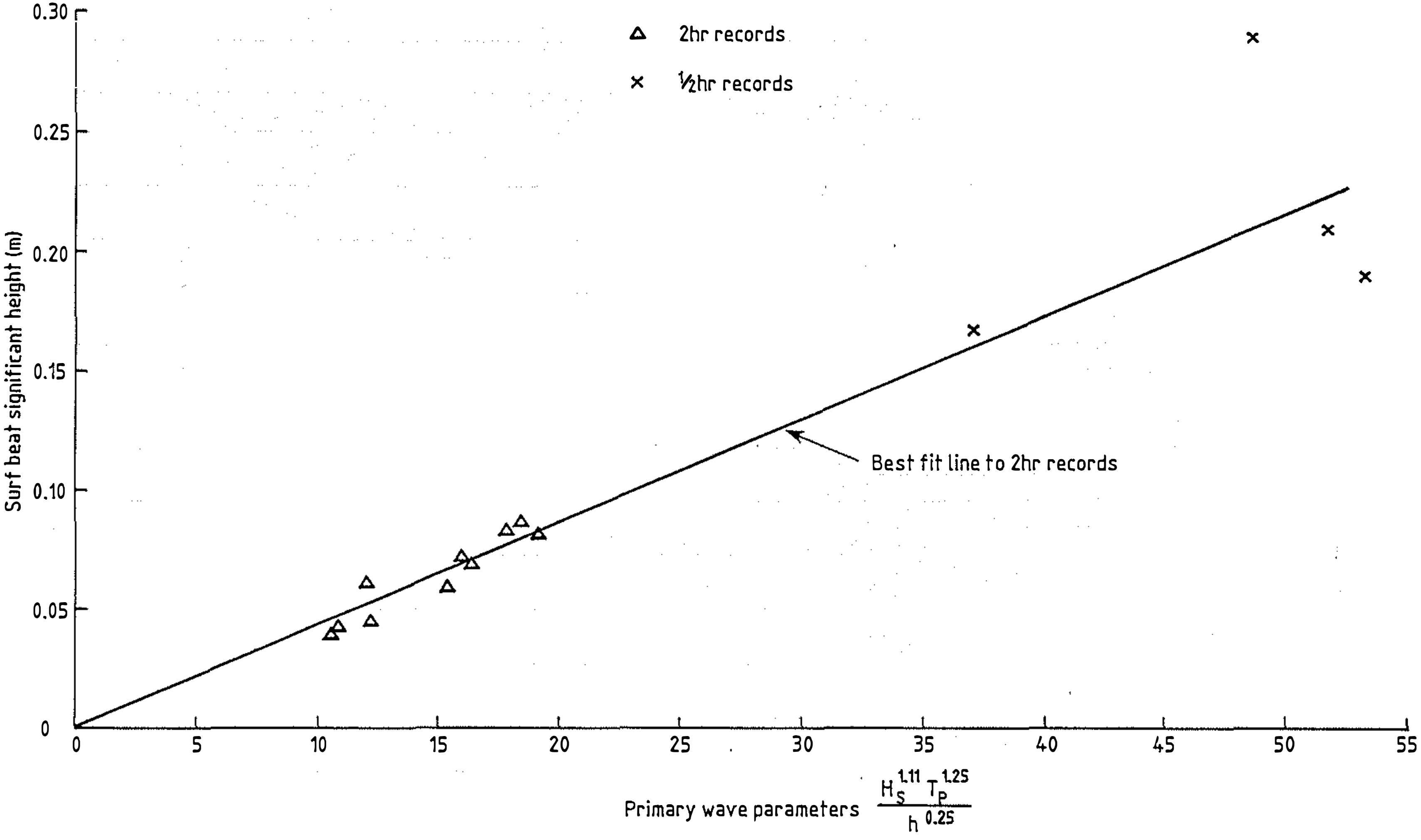
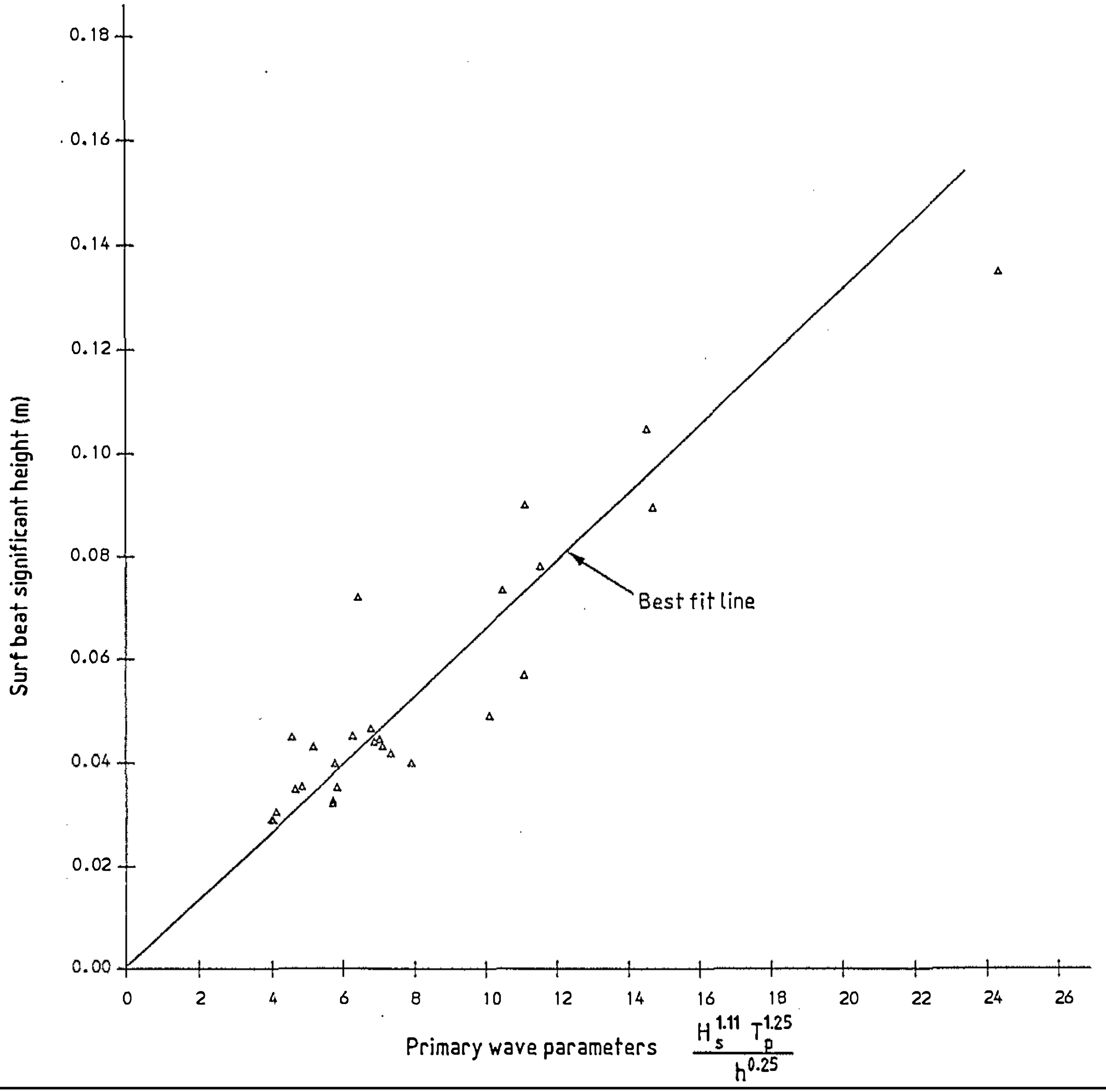


Fig 9 Surf beats off Shoreham



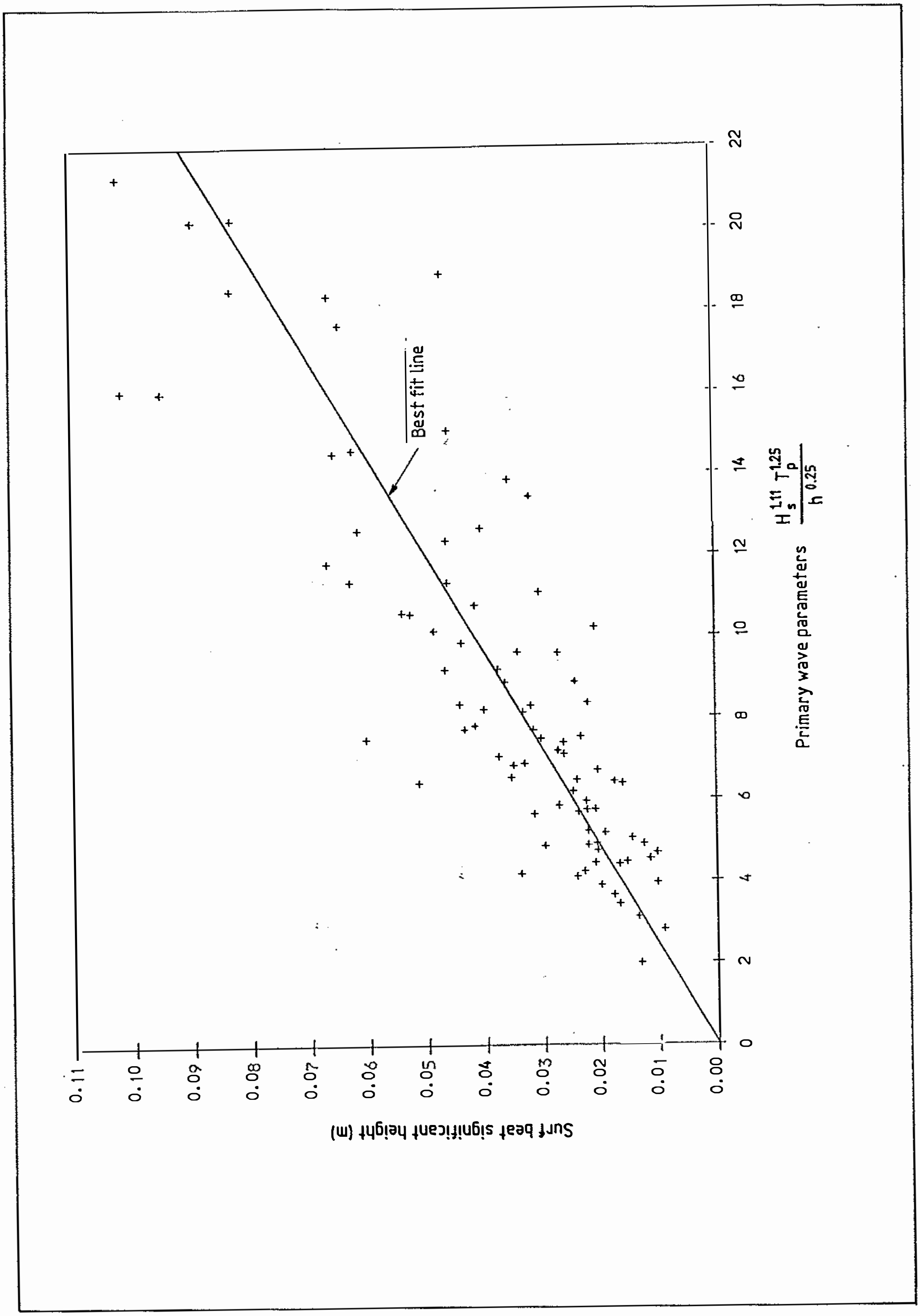


Fig 10 Surf beats off Barrow-in-Furness

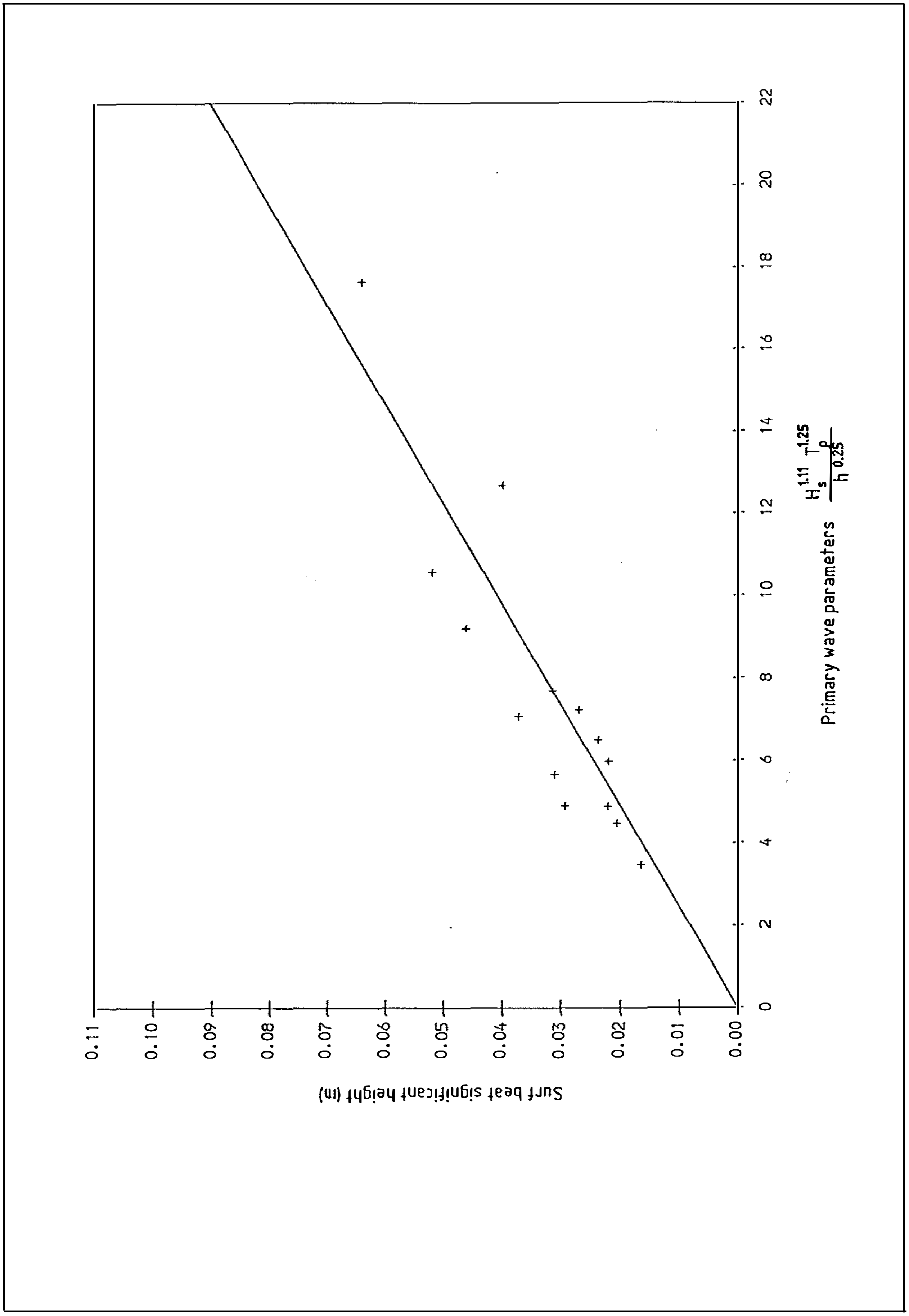


Fig 11 Surf beats off Barrow, 180°-200°N

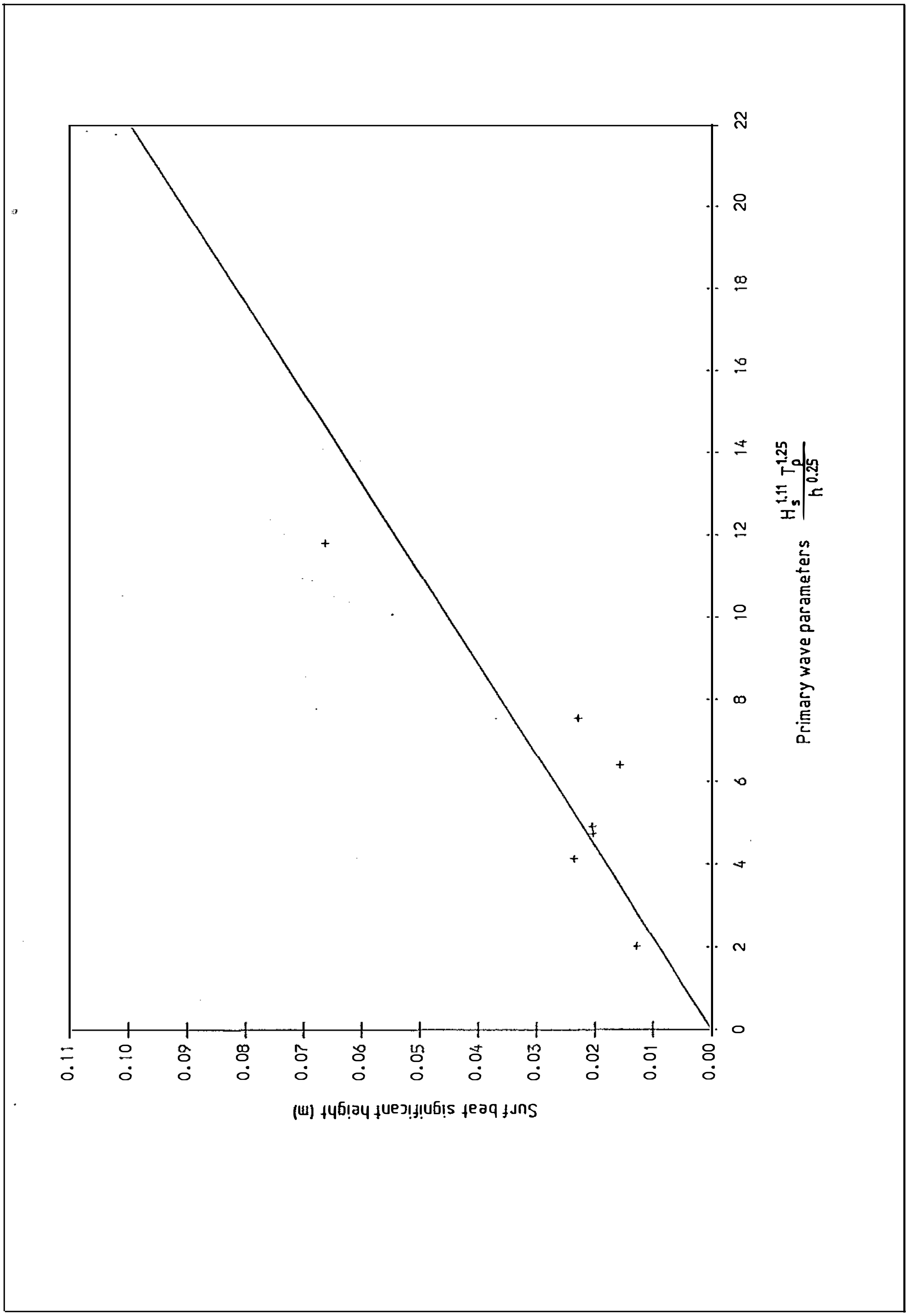


Fig 12 Surf beats off Barrow, 200°-220°N

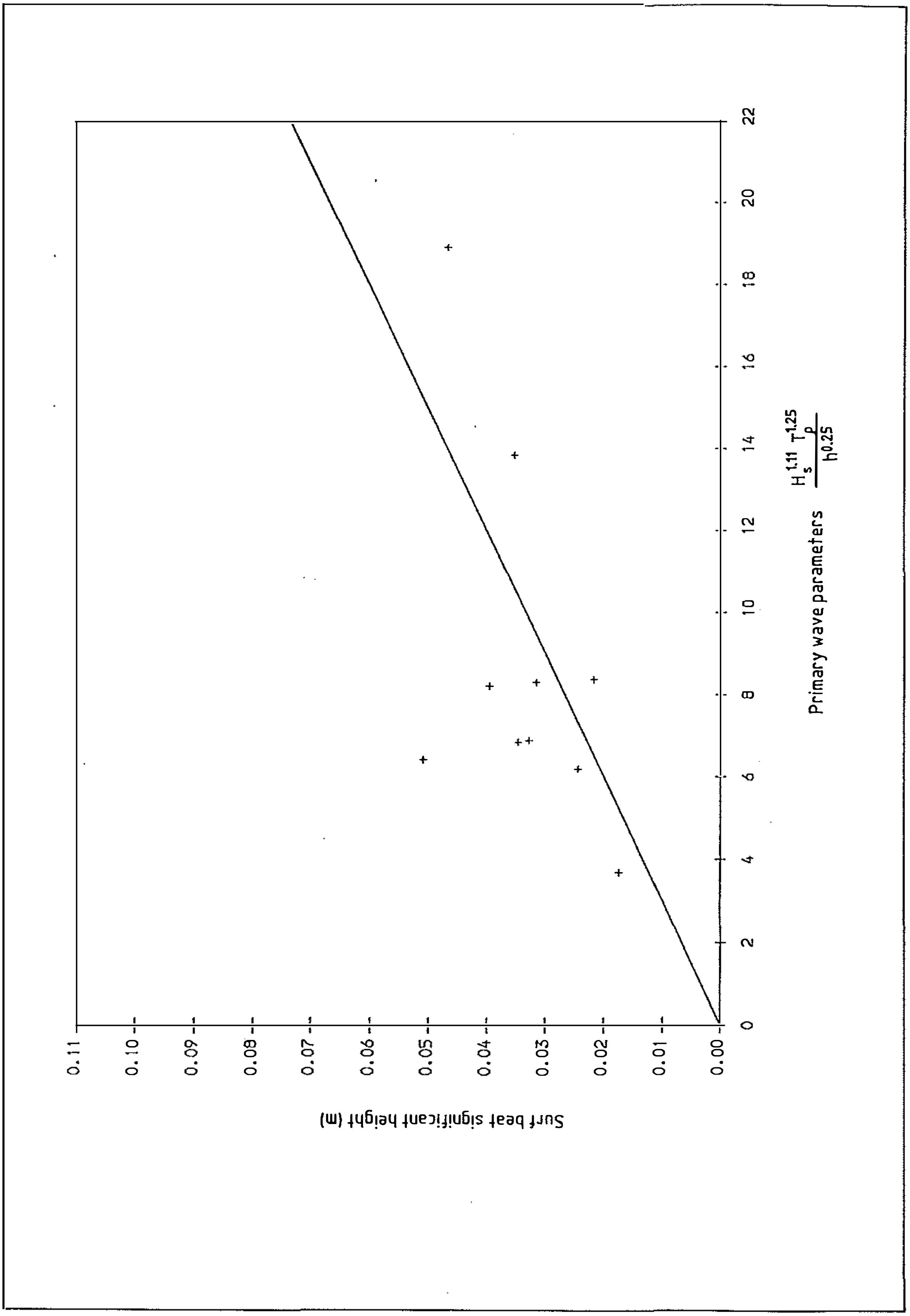


Fig 13 Surf beats off Barrow, 220°-240°N

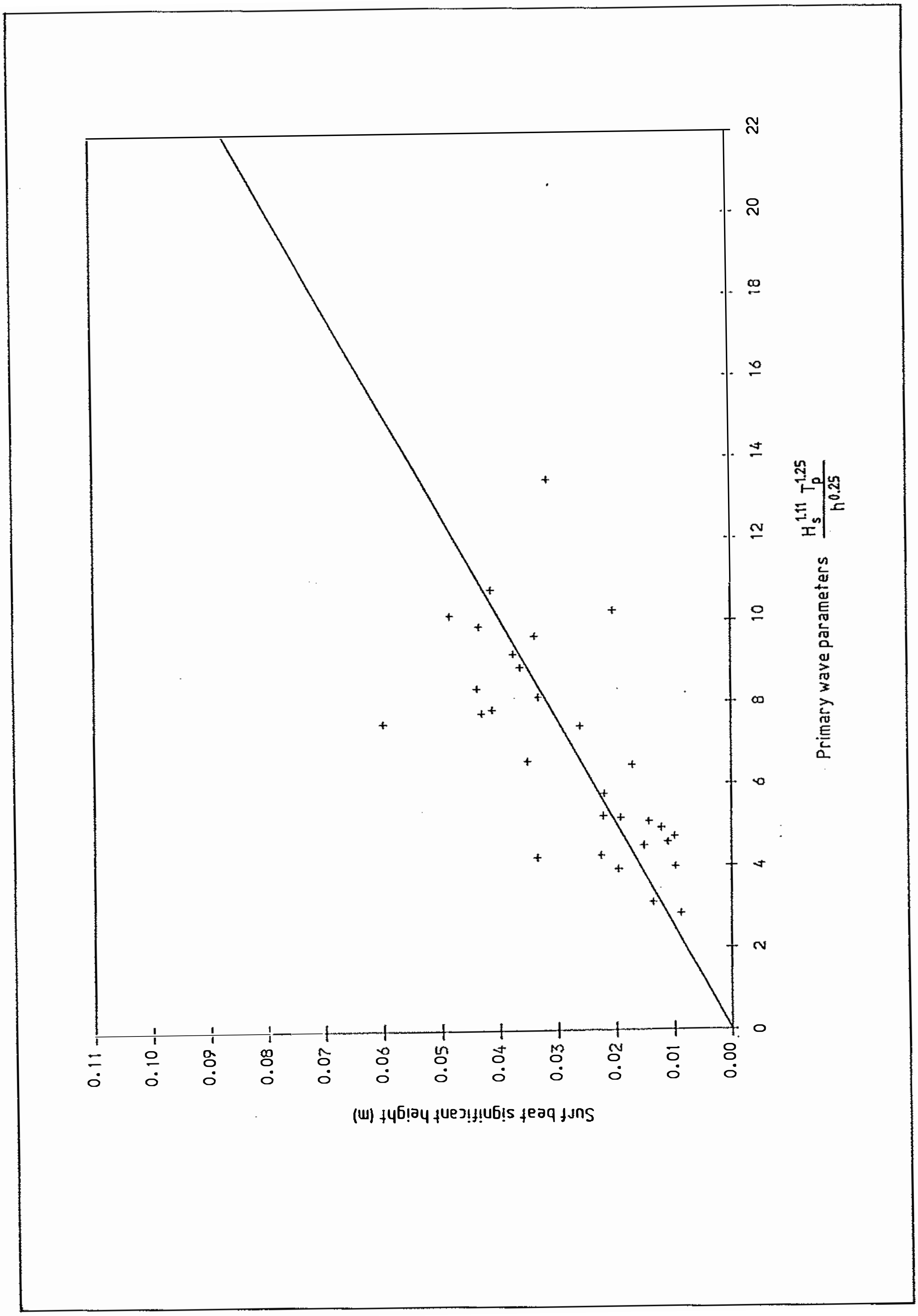


Fig 14 Surf beats off Barrow, 240°-260°N

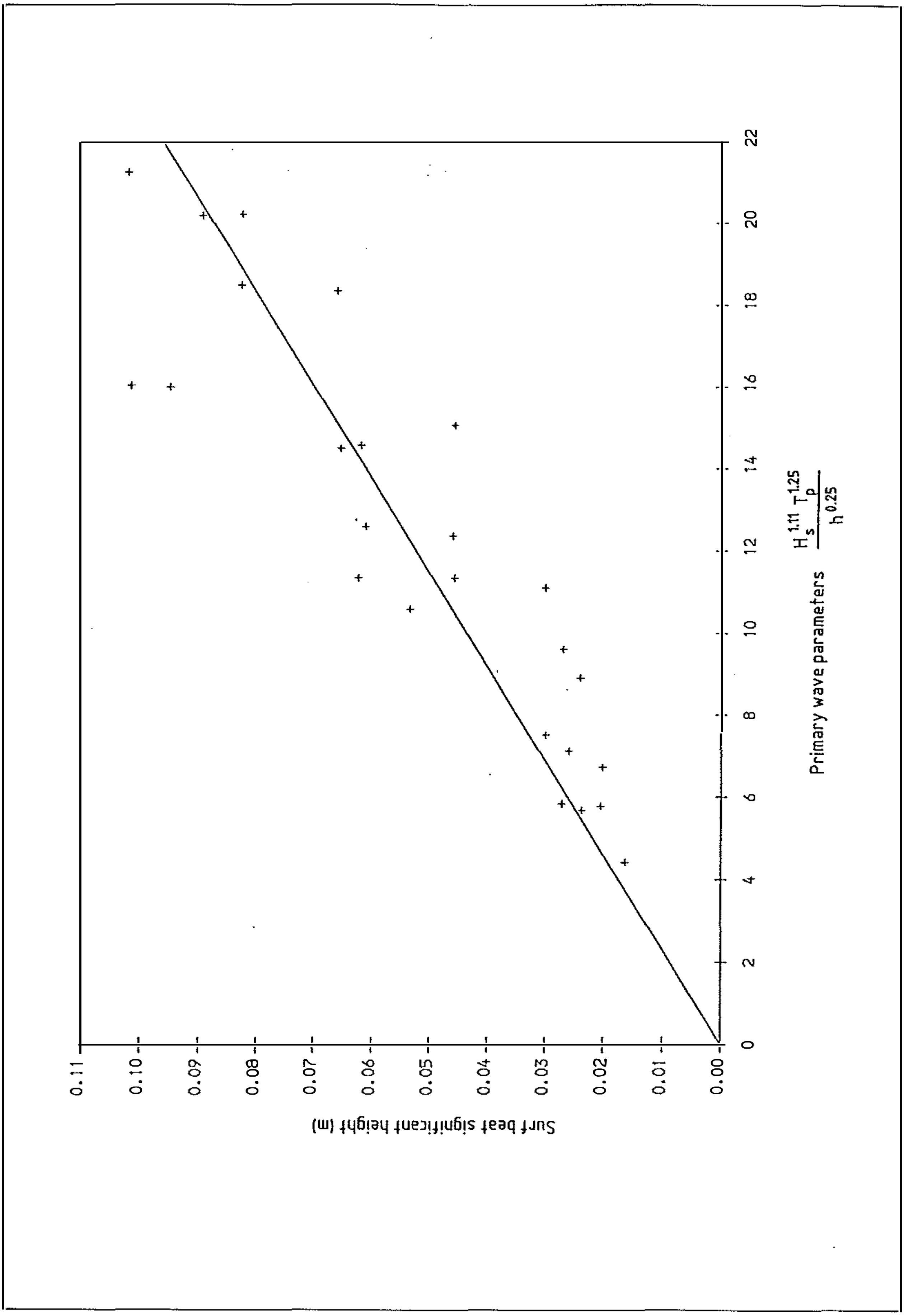


Fig 15 Surf beats off Barrow, 260°-300°N