DEVELOPMENT OF PATCHED TRANSPORT MODELS FOR ESTUARIES USING AN ICL DAP

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ABSTRACT

Depth integrated models for the transport of heat or mud in estuaries have been developed that use the ICL DAP at Hydraulics Research Ltd. These models incorporate areas of different size grid (or patches) so that they can be used with the results from a patched depth integrated DAP flow model.

When an upstream differencing algorithm for the transport is used these DAP models are found to give identical results to previous serial versions of the models. When flux corrected transport is used to give more accurate simulation of the movement of heat the results of the DAP and serial models are not the same but they are found to be very similar when compared to the results using upstream differences.

The DAP models are very advantageous in terms of speed if either a very small grid is used or if simulations of long periods of time are needed. The tests carried out also emphasize the importance of using flux corrected transport (or some other scheme) to minimise the numerical error obtained with upstream differencing.

This work will make the DAP available for a wide range of types of projects (pollution, ecology, environmental, thermal recirculation and sediment transport) undertaken by Hydraulics Research Limited.
1 INTRODUCTION

Very often the aim of a study is to predict the movement of heat, mud or pollution in an estuary. To this end the flow is usually simulated first, assuming that it is not affected by the substance being transported. If stratification is not important depth integrated flow and transport models are used. Once the flow pattern is established it is possible to test different pollution loadings, outfall positions etc, by running the transport model always on the same flow data. The results from the flow model are stored in the form of discharges through the faces of the cells and elevations at the centres of the cells at regular intervals (usually 10-20 minutes) during the tide.

The flow data is then read in to the transport model together with data on pollution loading, decay rates etc in order to compute the movement of pollution, head or mud. An advantage of this approach is that although a very small timestep may be needed in the flow model (perhaps a few seconds) a quite considerably larger timestep (of the order of ten times as large) can be used in the transport model. For this reason one tide of the flow model is usually much more expensive than one tide of a transport model. However it may be necessary to model the transport of pollution or mud for many tidal cycles to find a repeating pattern representing the actual conditions in the estuary.

It was thought to be useful to have DAP transport models for mud and heat. Although these models are often rather quick and cheap, for models with many cells or models with very small gridsize and timestep or when many tides are required the speed advantage of the DAP may prove useful.

These models are part of the Hydraulics Research TIDEWAY system for predicting the effects of engineering works on conditions in tidal waters. After obtaining the water flows using TIDEFLOW-2D the transport model can be used with mud, heat or ecological options to simulate the required quantity.

2 PATCHING ON THE DAP

Patching a model means having areas of different size grid (differing in cell size by a factor of 3). By this means the study zone can be resolved with a fine grid but a coarser grid may allow the boundary conditions to be applied well away from the region of interest. Further details are given in Refs 1 and 2. The development of a patched transport model allows
heat transport to be modelled using the results of the existing DAP patched flow models.

The ICL DAP (Distributed Array Processor) comprises 4096 arithmetic processors in a 64 square array which can all carry out the same instructions simultaneously on 4096 sets of data. The storage available comprises 4096 bit planes (see Fig 1), of which each real or integer matrix occupies 32 planes. The way in which Hydraulics Research applies the DAP to hydraulic studies is described in Ref (3).

Patching in the transport model is implemented in a similar way to in the flow model. In an area where flow is from the fine grid into the coarse grid then the heat flux into the coarse grid is the sum of the fluxes leaving the fine grid. So the total flux leaving the fine grid has to be moved by a patching subroutine into the coarse grid. The opposite is true where the flow is from the coarse grid into the fine. As in the flow model (Fig 2), the data is transferred by setting up a logical mask or matrix in the DAP to define a one to one mapping between the fine grid cells, on one side of the patch and the coarse grid cells on the other side. Fig 2 shows this process for the y fluxes.

The coarse cells shown in grid 1 are assumed to be those that butt on to the north of the fine cells shown in grid 2. The logical mask thus formed can be readily used to transfer information between the patch boundary cells shown in grids 1 and 2. The direction of transfer of information is from grid 1 to 2, for southward flow velocities and grid 2 to 1 for northward velocities. Associated with this process will be the transfer of x fluxes between the same two grids.

The models have this process systematised for up to 3 grids of size 64 by 64. If some grids are larger than this then joins between chunks of grid of the same size will be needed as outlined in Ref 3.

3 HEAT AND MUD TRANSPORT MODELS

The transport of many substances in estuaries can be modelled using depth integrated models. They include organic and inorganic pollution (including interactions), heat, mud, salt and sand. Particularly important applications of the present model are likely to be for modelling heat and mud transport.

It is important when designing power stations using once-through cooling water to consider the fate of the
rejected heat. Its possible effects include the impact on ecology, the warming of bathing water and recirculation. In modelling heat in a depth integrated transport model, it is the background heat that is being considered (as initially the heat is not distributed evenly through the vertical and does not move with the mean water speed). This is a part of the heatfield where the loss of heat to the atmosphere is important and the heat field may cover a huge area (albeit at very small temperature elevation above ambient) so the model usually needs to cover a large area. As there is very little in the way of source or sink for the heat it is particularly important to model the transport accurately and for this reason flux corrected transport is used (see below).

In the case of mud transport erosion and deposition tend to be the dominating features and it is possible in some cases to use the less accurate upstream differencing method. This is quicker and more robust than flux corrected transport. However it is important to model correctly the consolidation of the bed and the rate at which the mud bed is eroded, during the run of the tide. Although the physics of these processes is not understood well enough to enable precise predictions of siltation and erosion to be made the models do provide valuable information for the engineer to assess the impact of schemes.

4 FLUX CORRECTED TRANSPORT

The usual method used by Hydraulics Research for calculating heat transport in estuaries is to use the flux corrected transport algorithm (Ref 4). The first part of this technique is to do a timestep using upstream differences, that is the flux of heat across each cell face is taken to be the product of the water discharge and the concentration in the cell from which the water is coming. It is clear that this scheme tends to smooth out variations. In one dimension the numerical diffusion is of the order of \( \frac{3}{2} u \Delta x (1 - u \Delta t / \Delta x) \) where \( u \) is the velocity and \( \Delta x \) and \( \Delta t \) are the space and time steps. If \( u \Delta t = \Delta x \) any variation crosses a cell in one step and the error is zero. However, if \( u \Delta t \) exceeds \( \Delta x \), the scheme involves negative diffusion and is unstable. Hence as \( u \) is not constant in a practical case, there will be places where very large numerical diffusion of order \( \frac{3}{2} u \Delta x \) will occur to smooth out fluctuations.

The method of flux corrected transport is to correct the flux to be equal to the discharge of water multiplied by the average concentration of the two cells between which the water is passing.
Unfortunately if this is applied throughout there is no numerical diffusion, but spatial oscillations occur instead. The method therefore consists of correcting the flux as far as possible without introducing any new maxima or minima into the solution. This cuts out the oscillations and also reduces the spurious diffusion which now only exists in sensitive regions.

Such a scheme has been used for a while in serial programs but its use with the DAP cannot be exactly the same, because the serial technique employs a sweep through the grid. At each cell the x and y fluxes are corrected before proceeding to the next cell. The DAP program does the same but the x fluxes are corrected for all the cells and then the y fluxes so the result is not exactly the same as in the serial case. Tests are described below comparing calculations with the DAP and a serial machine both with upstream differencing and with flux corrected transport.

5 TESTS AND TIMINGS

5.1 The model geometry

The patched model geometry used for the tests was a 2D depth integrated, 2 grid representation of the Severn Estuary and Bristol Channel, Fig 3. The grid sizes are 4500m and 1500m. This model is used only as an example of a realistic flow field and was not calibrated to agree with nature.

5.2 Test with a step in temperature

A standard test is to look at one dimensional uniform flow with a channel initially full of water at a fixed temperature. The temperature at the end is changed and in the absence of any diffusion a step change in temperature should propagate along the channel. Such tests are described in Ref 5.

Upstream differencing is found to smooth out the step and centered differencing gives rise to a lot of oscillations. Flux corrected transport improves the results considerably although the step is still diffused to some extent.

In real cases there is usually a tide so that a step propagates both forwards and backwards. Differential motions cause the step to smooth out even without any diffusion (strictly speaking thin filaments form and small scale diffusion eventually acts to smooth these filaments out).

An interesting test is therefore to run for several tides with one temperature inside the model and
another on the boundary. This test was carried out using upstream differencing and flux corrected transport with both serial and DAP programs. It was found that in both cases the serial and DAP models gave very similar results. The temperature along the line $L$ shown in Fig 3 after the completion of ten tides is shown in Fig 4 for upstream differencing and flux corrected transport. This test was also carried out with a different timestep and the result was almost indistinguishable.

Using upstream differences the effect of the boundary temperature propagates up the model beyond the patch line and the step is very much smoothed out. With flux corrected transport on the other hand the step is confined to only about 3 cells. There is a little overshoot at the top but towards the boundary the temperature is more accurately zero.

The test demonstrates that with upstream differences there is a smooth transition at the patch. With flux corrected transport the temperature gradient has not reached the patch after 10 tides. Most importantly the diffusion that results from using upstream differences (here it is of the order of 1000 m$^2$/s) is quite destructive of the true solution when advection is the main influence on the heat field. Judgement is needed when numerical diffusion is smaller (e.g. when the grid size is reduced) or when source and sink terms are more important than advection (e.g. in mud transport modelling) but it seems likely that upstream differencing will usually give unacceptable results.

5.3 Test with a discharge near to the patch

A discharge was modelled at point $D$ in Fig 3. This was to investigate problems associated with the flux corrected transport algorithm near to the patch line as it is not readily possible to incorporate the flux correction across the patch line. The test was first carried out for a period of one tide starting at high water. The result in Fig 5 shows the temperature along the line shown in Fig 3 at high water. With flux corrected transport there is a peak at the outfall and another to the east which is due to hot water discharged at low water slack. This often observed phenomenon is found to be completely washed-out in the result with upstream differencing showing how poor an approximation it is in this particular case. To look at the results at the patch, as the discharge is on the landward side of the patch the model was ran for a further $\frac{1}{2}$ tide to low water so that the plume then extends over the patch.
Figs 6 and 7 show the contours of temperature at this time. The version with upstream differencing shows much more nearly continuous contours at the patch as the flux corrected transport algorithm is not satisfactorily handling the advection near to the patch. It seems wise in these circumstances to make sure that patch lines are placed well away from any outfall in a region where temperature gradients are less steep and less error will occur.

5.4 Model timings

The cost of a DAP model is made up of two parts, the DAP time (calculating) and the host machine time (inputting and outputting to filestore and to the DAP). The sum of these two is to be compared with the total time taken by a serial program. As the DAP is extremely fast for computation the advantage of using the DAP is greatest if a great deal of DAP arithmetic can be performed between successive appeals to the host machine. With a transport model this means if a large number of steps can be performed between successive storage times of the flow model. This would be the case if the model has a very fine grid (and consequently a very small flow model timestep).

For the tests described above the ICL 2972 computer used about .72 sec per step with no flux corrected transport and about 1.2 sec with flux corrected transport used throughout. The DAP used about .03 sec per step without flux corrected transport and about .05 sec with flux corrected transport. The DAP program also used 13 seconds of ICL 2972 time in calculating 1 tide. It can thus be clearly seen how great an advantage there is in using the DAP even in this case where only 1347 points are used (up to 8192 points could be used for the same DAP computing time).
6 CONCLUSIONS

Patched, DAP depth integrated heat and mud transport models have been developed for use with the data from patched flow models. These models show the greatest advantage over serial models when it is either necessary to simulate mud transport for a long period of time or if a model has a small gridsize so that the timestep of a transport model is small (e.g. a gridsize of 10m would imply a timestep of only a few seconds).

The results of the models have been compared for a realistic flow pattern and found to give results in reasonable agreement with the presently used serial programs. In the process it is shown how important it may be to include flux corrections especially if the grid is large. It was also found that the results were insensitive to the transport model timestep.

These models should be a most useful addition to the Hydraulics Research TIDEWAY-2D system of estuary flow and transport models.


Fig 1  Schematic diagram of D.A.P.
Fig 2  Patching on the D.A.P. (patching V velocities)
Fig 3 Flow model used for tests
Fig 4  Model results with step in temperature (after 10 tides)
Fig 5  Model results with point discharge (after 1 tide)
FIG 6  TEMPERATURE CONTOURS WITH FCT
FIG 7    TEMPERATURE CONTOURS WITHOUT FCT