THRESHOLD CONDITION OF SAND PARTICLES
UNDER CO-DIRECTIONAL COMBINED WAVE-AND-CURRENT FLOW

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ABSTRACT

Knowledge about the movement of sediment on the sea bed is highly important for the design of coastal structures and dredged navigation channels. In many cases sediment on the sea bed is moved by the combined influence of waves and currents. But so far the work done on the threshold condition of sediment movement under combined flow is rather limited because of its complex turbulence structure.

In the present report some results obtained by laboratory flume tests have been given. Experiments have been carried out on rippled and flat beds with five different sizes of sand. The bed shear stress over a rippled bed has been calculated recently from the measurements using the proposed method of Fredsoe. Reynolds stress measurements have also been made to obtain the bed shear stress over a flat bed.

An important result is that the Shields' curve can be used to determine the threshold condition under combined flow as well as under uni-directional and purely oscillatory flow, provided that the bed shear stress can be calculated. However, the maximum bed shear stress must be used instead of the mean bed shear stress when considering the threshold condition. The maximum bed shear stress on a rippled bed is taken as the maximum skin friction on the rippled crests, which is where sediment movement first occurs. All bed measurements for both flat and rippled bed condition are in good agreement with the Shields' curve.

This result enables predictions of bed mobility to be made for engineering problems in locations where currents and waves are both important, thus extending previous methods which could only be applied to locations in which either wave effects or current effects could be neglected.
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The combined flow of waves and current is a commonly occurring condition in the ocean. Sediment at the sea bed may move under the combined influence of waves and current, when it would not move under the same current on its own. Once the sediment is moving the action of the waves magnifies the sediment transport [1]. Although waves are efficient at disturbing sediment, they will only provide a net transport if there is a non-zero mean current, which may or may not be produced by the waves themselves.

As is well known, the threshold condition for bed movement under unidirectional flow is given by Shields' curve. Under pure wave conditions, various (≈ 20) formulae are available to estimate the threshold condition, which may give widely different answers. Sleath [2] shows that the best agreement with data is found by calculating the maximum bed shear stress and using Shields' curve. It had been thought that the acceleration effects under waves were an important factor in determining the threshold, but this is evidently not so [3, 4, 5, 6, 7, 8, 9].

Previous studies of the threshold condition under combined wave and current flows have been made by Inman [10], Hammond & Collins [11], Natarajan [12], Arafa [13]. However, their results are not easy to use for practical purposes, because wave and current velocities were used instead of the maximum bed shear stress in order to estimate the threshold conditions.

In practice a naturally occurring sea bed is more often rippled than flat. This further complicates matters, because the rippled bed has an additional resistance to the flow due to the form drag on the ripples. The bed shear stress on a rippled bed thus consists of the sum of the form drag and skin friction. Only the skin friction affects the grains
and can cause sediment motion. Over a flat bed the total bed shear stress is equal to the skin friction. Bagnold [14] pointed out (without experimental justification) that the total bed shear stress needed for sediment motion under unidirectional flow over a rippled bed is several times the skin friction, for grains finer than 0.9mm. Kapdasli & Dyer [15] found in laboratory experiments that the total bed shear stress at threshold under unidirectional flow condition is 8 - 12 times the skin friction. It has also been pointed out by several researchers that under oscillatory flow the critical total bed shear stress for sediment movement is considerably higher than the skin friction [16], [17] and [18].

In the present work, laboratory experiments have been carried out to observe the threshold conditions under combined wave and current flows on rippled and flat beds, and the threshold conditions have been estimated in terms of the maximum bed shear stress. Only co-directional combinations of waves and currents have been used.

2 CALCULATING BED SHEAR STRESS

Under unidirectional flow, the mean bed shear stress is usually used to determine the threshold condition, although the sediment only moves intermittently at times of peak bed shear stress due to bursts. There may be some sediment motion called 'weak movement' occurring at shear stresses less than the Shields' value [19], [20]. But in order to observe movement below the critical condition one must wait for a long time because of the statistical character of the turbulence in uni-directional flow.

Under an oscillatory flow the maximum bed shear stress must be used to determine the threshold of motion. The bed shear stress can exceed the critical value as often as four times during the wave period, but for
the rest of the period it is well below the critical value [21]. Because of the regular occurrence of the critical condition, under oscillatory flow the motion of the sediment is more clearly defined and it is easier to observe the threshold condition. Under a combined flow, the bed shear stress changes during the wave period, but at the same time bursts may also occur due to the current. For this reason the maximum bed shear stress must be used to determine the threshold of motion.

For pure wave or pure current conditions, the total bed shear stress can be calculated by considering the turbulence properties well away from the bed. However, this is more difficult under a combined flow because it has a very complex turbulence structure, in which there is a thin turbulence boundary layer which mainly depends on the wave motion, while outside the boundary layer the turbulence structure is mainly created by the current [22]. Measurements made by Kemp & Simons [23] showed that the maximum bed shear stress on a smooth bed over a wave period is approximately twice the average bed shear stress, for a large wave. Over a rough bed the turbulence intensity increases considerably when even a small wave is added to the current. In this case turbulence fluctuations are very large. This means that it is easier to reach the threshold condition under the combined wave and current flow than under a current alone.

Several theoretical methods have been proposed to obtain the velocity distribution and bed shear stress under a combined flow (Lundgren [24], Bakker [25], Bakker & Doorn [26], Grant & Madsen [27], Bijker [28], Smith [29]), and measurements have also been made by George & Sleath [30], Brevik & Aas [31], Brevik [32]. A recent method developed by Fredsøe [22] is used in the present study to calculate the maximum bed shear stress, because it is the easiest one to use in the
present experiments to obtain the maximum bed shear stress from the measured quantities.

In this theory it is assumed that there is a turbulent wave boundary layer due to the combined flow of wave and current, above which the turbulence is subject only to the mean flow. With these assumptions, different velocity distributions are considered inside and outside the boundary layer.

Over a rippled bed the skin friction at the ripple crests is taken as the critical bed shear stress which is calculated by the Fredsoe method in the present work, because the maximum skin friction occurs near the ripple crests, where the first sediment motion occurs. Previous work [15] under a pure current showed that the skin friction at the ripple crests on threshold condition was equal to the Shields' value.

An alternative method of obtaining the bed shear stress is via the Reynolds stress. This can be calculated by measuring the vertical (w') and horizontal (u') fluctuating components of water velocity near the bed, from which the stress $\rho u'w'$ can be calculated.

If the height at which the velocity components are measured is quite near the bed it is valid to assume that the Reynolds stress is approximately equal to the bed shear stress [17]. However it is very difficult to obtain an accurate estimate of the Reynolds stress over the rippled bed, because the velocity components are not aligned vertically and horizontally because of the local bed slopes, and small alignment errors cause large errors in calculation of the Reynolds stress. Therefore this method has been used only over the flat bed.
3 EXPERIMENTAL METHODS

The laboratory experiments were carried out in a standard 34m long, 0.60m wide and 0.6m deep recirculating flume at the Hydraulics Research laboratories. A piston type wave maker was used, located at the inlet end of the flume. The current entered horizontally through a gap of 10cm under the wave paddle. The current velocity was changed with a valve located after the pump.

In the experiments three wave periods of 1.5, 1.8 and 2.0s were used, with at least 2 different wave amplitudes for every wave period. For simplicity, the water depth was kept constant at 23cm in order to eliminate any depth dependency in the results.

At the downstream end of the flume a sloped porous beach was located to absorb the propagating waves and to provide an undisturbed flow in the channel.

The wave amplitude was measured by means of a wave probe which was situated above the sand bed and was recorded on a chart recorder. The wave-probe was calibrated frequently.

In order to measure the near bottom velocities at the ripple crests, a two component laser doppler velocity meter was used, which was focussed over the sand bed via windows in the sides of the channel [33]. The vertical and horizontal components of water velocity were recorded on the same chart as the wave amplitude. An important point was to arrange the laser velocity meter to be focused just above the ripple crests where the sediment first begins to move. This arrangement was made at the beginning of every set of experiments. The near bottom velocities were measured at 0.7cm above the ripple crests, because with the existing channel facilities it was impossible to measure at a closer point to the rippled bed.
The vertical distribution of velocity was measured by using a calibrated \text{1cm} diameter electronic micropropeller. The measurements of pure wave motion were made by using a two component electro-magnetic current meter. Both of them were fixed to a beam at the top of the channel with a scale that could be moved vertically to measure the velocity at every point. Velocities were measured at \text{1cm} intervals from the ripple crest to the surface, and the measurements were repeated several times. Average velocity measurements were taken over 20 seconds, and maximum velocities were observed over 30 seconds.

A sand bed of \text{5cm} thickness and \text{5m} length was located in the middle of the channel where fully developed combined flow occurs. Five different sizes of uniform sand were used in the experiments to obtain the variation of threshold condition with grain diameter. Their diameters were \text{137\mu m}, \text{220\mu m}, \text{370\mu m}, \text{550\mu m} and \text{650\mu m} which are all suitable for producing ripples at the sea bed.

In each set of experiments for a particular grain size, water was gently poured up to \text{23cm} without disturbing the sand bed. The water temperature was \text{20 \pm 2^\circ C} for all the experiments.

For the rippled bed experiments, in order to obtain the rippled bed the wave was started over an initially flat bed and gradually increased until fully developed ripples covered the whole of the sand bed surface. After that, the wave gradually decreased and stopped, and still water was obtained without disturbing the ripple pattern. The ripples were generally two-dimensional, with different sizes depending on the sand grain diameter.

Every test was begun with a pure oscillatory flow which was insufficient to create sediment motion. After measuring the wave period, wave amplitude, near
bottom velocities and velocity distribution under the pure oscillatory flow, the combined flow was created by applying a current to channel. The current was gradually increased until the threshold condition was obtained, and all measurements were made under the combined flow at the threshold condition. Finally, the wave was stopped and allowed to settle until undisturbed unidirectional flow was present. Velocity measurements were made under the unidirectional flow, which alone was insufficient to cause sediment movement. Between any two tests at least half an hour was allowed to obtain still water in the flume.

The threshold condition which was assumed was that approximately half of the grains are moved over the whole flat bed, or at the ripple crests on the rippled bed. The observation and determination of the threshold condition under combined flow is rather difficult, because it depends on the combination of wave and current. If the superposed current is weak so that there is a wave-dominated environment, the grains at the ripple crests begin to move quite suddenly when the threshold condition is reached, as under pure oscillatory flow. In this case it is comparatively easy to determine the threshold condition. When the current is strong, before reaching the threshold condition a few grains may move very weakly, as under current alone. In this case the current must be increased very gradually to provide an accurate assessment of the movement of sediment. A lot of preliminary tests were done to ensure consistency of judgement of the threshold. The results obtained in the experiments indicate that the determination of the threshold condition was quite adequate.

The threshold condition of sediment motion on the flat bed was calculated in terms of the Reynolds stress measurements. For this purpose the $u'$ (horizontal) and $w'$ (vertical) fluctuating components of water
velocity 0.6 cm above the bed were measured by passing the signals from the laser system through high-pass filters. The product of $u'$ and $w'$ was obtained from a multiplier unit whose output was passed through a low-pass filter to obtain the mean product $U'W'$, which was recorded on a chart recorder. All the filters were set to a time constant of 20 seconds. To ensure that the fluctuations were not affected, the focusing was checked several times in every experiment.

4 RESULTS

4.1 Threshold velocities over a rippled bed

The variation with grain diameter of the maximum bottom velocity obtained from the laser time series of horizontal velocity just above the ripple crests is given in Figure 1. It is shown that the maximum bottom velocity under pure oscillatory flow at threshold is about 30% less than that of pure unidirectional flow. This is because the boundary layer is thinner under waves so that there is greater shear stress near the bed.

The results for the combined wave and current flow lie between those for the pure wave and the pure current, and depend on wave amplitude and period. It was observed that the critical maximum bottom velocity under the combined flow decreased with increasing wave amplitude or decreasing wave period. Thus it is quite difficult to determine the threshold condition under combined flow by using flow velocities alone.

Vertical distributions of the maximum velocities at threshold are shown in Figure 2. The maximum velocities reach a constant value at a height of about 11 cm above the bed, indicating that the water depth should not affect the results. The velocities exhibit a minimum at ~3 cm, probably due to speed-up
immediately above the ripple crests \[^{[34]}\]. It can be seen that the velocities increase as the grain size increases.

The maximum and average velocity distributions at threshold for the 550μm sand, are given in Figure 3, together with the current-alone velocity distribution which on its own was insufficient to cause sediment movement. This shows that a superposed wave can create sediment motion by increasing the maximum velocities, but that the average combined velocity is smaller than the current-alone velocity.

### 4.2 Threshold skin friction on a rippled bed

The skin friction at threshold under combined flow was obtained by the Fredsoe method as follows. From the measurements we calculated the time-averaged depth-mean flow velocity \( V \), maximum bottom velocity \( U_w \) due to the wave, near-bottom orbital excursion amplitude \( a \), water depth \( D \) and bed roughness \( k \). The depth-mean velocity outside the boundary layer can be written as

\[
\frac{V}{U^*_C} = \frac{1}{k} \left[ \ln \left( \frac{300}{k_w} \right) - 1 \right]
\]

(1)

where \( k_w \) is an apparent bed roughness which is different from the conventional bed roughness because of the turbulent wave boundary layer, and \( U^*_C \) is the time-mean bed shear stress velocity. In the method, some iteration is needed to obtain \( U^*_C \) and \( k_w \). The first step is to choose an initial value of \( U^*_C \) and to obtain \( k_w \) by using Figure 7 of Reference 14, which shows the variation of \( k_w / k \) versus \( U_w / U^*_C \) and \( a / k \). If the \( k_w \) obtained from the figure is not equal to that obtained from equation (1), a new value must be chosen until agreement occurs. After obtaining the \( U^*_C \) and \( k_w \), the second step is to calculate the maximum bed...
shear velocity $\tau_{max} = \rho U_{max}^2$ by using Figure 13 of Reference 14, which shows the variation of $U_{max}/U_{c}$ versus $U_w/U_{c}$ and $a/k$.

The maximum bed shear stresses obtained by the Fredsoe method at threshold (Figures 4 and 5), are in good agreement with the shear stress curve and Shields' curve which are obtained under unidirectional flow. Thus Shields' curve can be used to predict thresholds for combined flow as well as for unidirectional flow. It further appears that the threshold condition does not depend on the wave period provided that the maximum skin friction is used in combined flow instead of the mean bed shear stress.

4.3 Threshold bed shear stress on a flat bed

The Reynolds stress measurement on the flat bed (Figure 6 and Figure 7) confirm the results of the rippled bed measurements; namely that Shields' curve gives threshold condition under combined wave and current flows on both flat and rippled beds. Again the results are independent of the wave period.

5 CONCLUSIONS

The threshold of movement of sand particles on rippled and flat beds under co-directional combined wave and current flows has been determined. Laboratory experiments were carried out with five different grain sizes and a wide range of variation of wave and current. The results obtained can be summarised as follows:

1. Shields' curve can be used successfully to determine the threshold of sediment movement over a rippled bed under combined wave and current flow, provided that the maximum skin friction is used instead of the mean bed shear stress.
2. Shields' curve also gives the threshold condition of sediment movement over a flat bed under combined flow.

3. The maximum bottom velocity of combined flow at threshold is affected by the wave period and amplitude.

4. It appears that the threshold of motion in terms of the maximum bed shear-stress does not depend on the wave period.

Thus it is possible to say that, despite the complex turbulence structure of combined wave and current flow, the threshold condition of sediment movement under combined flow is the same as under pure unidirectional and pure oscillatory flow.

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Fig 1. The max bottom velocity at threshold versus grain diameter.
Fig 2  Velocity distribution curves obtained by laboratory tests on the rippled bed under combined flow
Fig 3  Typical velocity distribution curves of max & average velocities of combined flow & the velocity distribution of superposed flow for a grain of 0.055 cm diameter
Fig 4  Threshold shear stress on rippled bed versus grain diameter under combined flow. Full line: the threshold shear stress obtained for flat bed under unidirectional flow (3)
Fig 5 Shields curve for a flat bed, broken line obtained for flat bed under unidirectional flow (3) \( (v = 0.01 \text{cm}^2/\text{sc}) \)
Fig 6  Threshold shear stress on flat bed versus grain diameter under combined flow. Full line: the threshold shear stress obtained under unidirectional flow (3)
Fig 7  Shields curve of flat bed, broken line obtained for flat bed under unidirectional flow $(3)(\nu = 0.01 \text{cm}^2/\text{sc})$