EstProc

Note on the use of algorithms for modelling mud transport on tidal flats

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J Spearman

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Summary

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1. **Introduction**

This work has been undertaken as part of the Estuaries Processes Research Project (EstProc). As part of this project a number of algorithms described in the EstProc final report (Estuary Process Consortium, 2004) have been developed relating to tidal mud flat processes. Algorithms of this kind are inevitably produced in a research environment and do not always prove useful in the research environment for various reasons:

- applicability (e.g. does the algorithm increase computer run time significantly?)
- relevance (does using the algorithms improve the results significantly?)
- data (is there real data from which to choose the parameter values?)
- sensitivity (does the algorithm make the results oversensitive to a particular parameter?).

The research development of an algorithm is therefore only a part of the process of the overall algorithm development. The algorithm is then tested in the “expert” community (peer review) to examine its overall usefulness as a process descriptor. This second stage of development can take a long time and it may be typically several years before the algorithm is available to the end-user for practical applications.

This piece of work sets out to highlight some of the applicability, relevance, data and sensitivity issues surrounding the (mud flat process) algorithms developed in the EstProc Project in order to kickstart the examination of their practical usefulness and provide some value-added algorithms to the end-user as a result.

2. **Selection of algorithms for testing**

The methodology chosen in this study has been to model the suspended sediment (mud) transport for a typical mudflat scenario from the simplest basis and to add complexity (ie add the algorithms developed in the EstProc Project) one by one. This process allows some identification of the issues of applicability, relevance, data and sensitivity. In addition, to allow the effect of the algorithms to be evaluated in the context of applied modelling, the sensitivity of the model results to using different numerical approaches such as 2D/3D modelling and the number of 3D layers used is also examined.

2.1 **WINTERWERP(2003) – ALGORITHM 1**

Winterwerp (2003) undertook a re-evaluation of the 1960’s experiments by Krone that led to the development of the widely used formula for the deposition of suspended cohesive sediment,

\[
\frac{\partial m}{\partial t} = \left(1 - \frac{\tau}{\tau_d}\right)w_s C
\]  

(1)

where \( m \) is the mass of sediment eroded, \( t \) is the time-scale, \( \tau \) is the shear stress exerted on the bed and \( \tau_d \) is the critical shear stress for deposition.
Winterwerp’s conclusion from the re-examination of the derivation of this formula was that this formula did not adequately reflect the observed laboratory measurements made by Krone. Winterwerp went onto conclude that Krone’s results actually suggested the following:

- There is no critical shear stress for deposition.
- Deposition on the bed occurs at a rate, \( w_s C \), regardless of the applied shear stress.
- Deposition occurs simultaneously with erosion.
- Changes in the bed are actually caused by net deposition or erosion.

Equation (1) therefore becomes,

\[
\frac{\partial m}{\partial t} = w_s C - M_e \left( \frac{\tau - \tau_c}{\tau_c} \right)
\]  

(2)

where \( m \) is the net (mass of) deposition occurring on the bed, the second term represents the rate of erosion, \( M_e \) is the Partheniades erosion constant, \( \tau_c \) is the critical erosion threshold.

This algorithm can be applied whether 2D or 3D modelling is being used.

### 2.2 EFFECT OF 2D/3D MODELLING – NUMERICAL APPLICATION 1

Though not strictly an algorithm the effect of using a 3D model was examined and compared to the results of the 2D modelling above. This consideration is more solution-orientated than physically based and sets a useful context in which to view the effects of including the algorithms listed below.

### 2.3 EFFECT OF TURBULENT DAMPING IN REDUCING BED SHEAR STRESS – ALGORITHM 2

The effect of turbulent damping is caused when the vertical density gradient in the flow becomes so large that the turbulent structure of the flow is affected. This has two consequences: the first is that the turbulent diffusion of sediment into the higher parts of the water column can be reduced (or if the density gradient is large enough, stopped entirely) leading to a further increase in density gradient; the second is that the shear stress exerted on the bed can be reduced leading to reduced erosion or enhanced deposition.

The first of these effects, the reduction in turbulent diffusion is reproduced in pretty well all 3D models. The second effect, that of drag reduction on the bed, is not included in all 3D models but as highlighted by Toorman (2000) this effect can lead to an over-estimation of the bed shear stress by a large factor.

Toorman (2000) identified that the boundary condition for the near bed flow (this has many forms but the one considered here is where \( u_* \) is eliminated and the derivative of the near bed current with respect to the vertical elevation \( \partial u/\partial z \) is expressed in terms of the current, \( u \)) should be given by,

\[
\frac{\partial u}{\partial z} = f_u \frac{u}{z \ln(\sqrt{u_*})}
\]

(3)
where \( u \) is the current speed “at the bed”, \( z \) is the height above the bed, \( z_0 \) is the physical roughness length and \( f \) is given by, \( f_u^{-1} = F_f f(\alpha) \). In this case \( F_f \) is the momentum damping function given by \( \nu = F_f \nu_0 \) where \( \nu_0 \) and \( \nu \) are the “undamped” and “damped” viscosities. \( f(\alpha) \) is a function which takes account density stratification on the logarithmic velocity law and \( \alpha \) is a parameter dependent on the settling velocity, Richardson number and friction velocity Toorman (2000). Toorman states that the effect of \( f(\alpha) \) is minor (the example he gives suggests typically of the order of 10%) compared to the effect of the turbulent damping on bed friction and so for this study \( f(\alpha) \) has been arbitrarily set to 1.

2.4 SOULSBY’S FORMULA FOR COMBINED BED SHEAR STRESS DUE TO WAVES AND CURRENTS – ALGORITHM 3

For the EstProc project Soulsby (Soulsby and Clarke, 2004) derived a formula for the combined bed shear stress due to waves and currents. The method as presented is derived on the basis of well-mixed systems where the bed shear stress can be reasonably estimated on the basis of the depth-averaged current and is therefore appropriate for 2D models. This derivation allows a more accurate (when compared to laboratory data), explicit (the formulae involved amount to a few lines, unlike other available algorithms which require numerical solution of implicit equations) and therefore simpler and quicker solution than other contemporary algorithms. The derivation is equally applicable to both rough-turbulent and smooth-turbulent systems.

For this study, Soulsby’s algorithm has been extended to the 3D case. The basis for the 3D algorithm is presented in Appendix 1.

The algorithm is compared against a simple model of combined stress, that of \( \tau_{w+c} = \tau_c + \tau_w \) where the subscripts \( w+c \), \( c \) and \( w \) signify combined shear stress due to waves and currents, currents alone, and waves alone, respectively.

2.5 MANNING’S ALGORITHM FOR SETTLING VELOCITY – ALGORITHM 4

Manning’s algorithm for settling velocity is an empirical formula, which though not presented in a dimensionless form, has the merit of being based on a large data set of accurate in situ settling velocity measurements from different estuaries and locations. The algorithm is based on the concept of macroflocs – large aggregate floc structures – and microflocs – representing the constituent particles of the macroflocs. Equations are given for the setting velocity of macroflocs, for the settling velocity of microflocs, the ratio of macroflocs to microflocs in the floc population and the settling flux (Manning, 2004):

\[
\text{MSF}_{\text{EM}} = \frac{1}{1 + \text{SPM}_{\text{ratioEM}}} \cdot \left( \text{SPM} \cdot W_{s_{\text{macroEM}}} \right) + \frac{1}{1 + \text{SPM}_{\text{ratioEM}}^2} \cdot \left( \text{SPM} \cdot W_{s_{\text{microEM}}} \right) \tag{4}
\]

Where \( W_{s_{\text{macroEM}}} \) (mm/s) is given by,

For \( \tau \) ranging between 0.04-0.7 N m\(^{-2}\):

\[
W_{s_{\text{macroEM}}} = 0.644 + 0.000471 \text{SPM} + 9.36 \tau - 13.1 \tau^2 \tag{5a}
\]
For $\tau$ ranging between 0.6-1.5 N m$^{-2}$:

$$W_{s_{\text{macroEM}}} = 3.96 + 0.000346 \text{ SPM} - 4.38 \tau + 1.33 \tau^2$$  \hspace{1cm} (5b)

For $\tau$ ranging between 1.4-5 N m$^{-2}$:

$$W_{s_{\text{macroEM}}} = 1.18 + 0.000302 \text{ SPM} - 0.491 \tau + 0.057 \tau^2$$  \hspace{1cm} (5c)

and where $W_{s_{\text{microEM}}}$ (mm/s) is given by,

For $\tau$ ranging between 0.04-0.55 N m$^{-2}$:

$$W_{s_{\text{microEM}}} = 0.244 + 3.25 \tau - 3.71 \tau^2$$  \hspace{1cm} (6a)

For $\tau$ ranging between 0.51-10 N m$^{-2}$:

$$W_{s_{\text{microEM}}} = 0.65 \tau - 0.541$$  \hspace{1cm} (6b)

and where $\text{SPM}_{\text{ratioEM}}$ (no units) is given by

$$\text{SPM}_{\text{ratioEM}} = 0.815 + 0.00318 \text{ SPM} - 0.00000014 \text{ SPM}^2$$  \hspace{1cm} (7)

The simulations results are compared with the results for a simulation with a constant settling velocity of 0.5mm/s.

2.6 EFFECT OF LAYER SPACING NEAR THE BED – NUMERICAL APPLICATION 2

Though not strictly an algorithm the effect of reducing the thickness of the 3D model layers near the bed was examined. The vertical resolution of the near bed has been shown in 1DV models to have a potentially significant effect on sediment transport (e.g. Winterwerp 1999) Moreover this consideration is more solution-orientated than physically based and sets a useful context in which to view the effects of including the algorithms listed above.

2.7 EFFECT OF BIOLOGY/SENSITIVITY TO SEDIMENT EROSION PARAMETERS

The sensitivity of the results to the choice of erosion threshold and erosion rate constant was also investigated. This was undertaken by considering the extent of the effects that biology could have on these parameters.

To examine the effect of biology two scenarios were selected to compare to the reference case (as given in Section 3.2). The scenarios were derived based on the work by Widdows in collaboration with other researchers as summarised in Chapter 4 of the EstProc Final Report (Estuary Process Consortium, 2004). The first scenario was selected to be a scenario were the biology produces an environment where the critical erosion threshold is relatively high and the rate of erosion is relatively low i.e. a very depositional environment. The second scenario was selected to be representative of biology producing a reduction in the erosion threshold and an increase in the erosion rate.
Widdows (2004) suggests a scenario whereby a high population of Microphytobenthos greatly enhances the cohesiveness and stability of the sediment. This is achieved through the production of mucus-like carbohydrates or EPS (extracellular polymeric substances). Microphytobenthos are present throughout the year ranging from 2 µg g\(^{-1}\) dry sediment in the winter to 60 µg g\(^{-1}\) in June. Widdows summarises the effect of the higher summer population densities as resulting in an increase in the erosion threshold of near bed (~10cm) current speed of 0.15m/s and a ten-fold reduction in erosion rate. This scenario was implemented in the model as a 0.1N/m\(^2\) increase in erosion threshold (from 0.2N/m\(^2\) to 0.3N/m\(^2\)) and a ten-fold increase in the erosion rate (which when the critical erosion threshold changes means a change from the reference Partheniades erosion rate of 2x10\(^{-4}\)kg/s/m\(^2\) to 3x10\(^{-5}\) kg/s/m\(^2\)).

In the same report Widdows suggests a scenario whereby a high population of Macoma could result in bioturbation and enhanced erosion of the bed. *Macoma* is a small clam that can occur at high densities (>10,000 individuals m\(^{-2}\)) in estuarine sediments ranging from fine mud to mud-sand mixtures. It is a surface deposit feeder, grazing on the microphytobenthos, thereby loosening the surface sediments, increasing bed roughness and water content. Widdows suggests that at medium to high densities the effect of Macoma is to reduce the critical erosion threshold of near bed (~10cm) current speed by 0.05m/s and to increase the erosion rate fourfold. A similar scenario was implemented in the model as a 0.05N/m\(^2\) reduction in erosion threshold (from 0.2N/m\(^2\) to 0.15N/m\(^2\)) and a two-fold increase in the erosion rate (which when the critical erosion threshold changes means a change from the reference Partheniades erosion rate of 2x10\(^{-4}\)kg/s/m\(^2\) to 6x10\(^{-4}\) kg/s/m\(^2\)).

2.8 SUMMARY OF SIMULATIONS UNDERTAKEN

Table 1 and Table 2 summarise the simulations undertaken and the algorithms and numerical applications which are involved in each simulation. The tables also list the figures where the results are displayed.

3. Methodology

3.1 INTRODUCTION

The chosen approach to modelling was to create a generic and typical mud flat upon which the chosen algorithms could be tested. The typical mudflat incorporated the following properties:

- water level variation
- onshore tidal current (a few centimetre per second)
- alongshore tidal currents with peaks varying up to around 0.3m/s at the edge of the mudflat
- suspended sediment carried onto the mudflat both from alongshore and from the adjacent channel.

The model was constructed so that the boundaries of the model were relatively coarsely represented but the internal mudflat elements were finely represented, effectively creating a profile model (1D or 2DV) of the mudflat.

Because of the potential complexity of examining the effects of these algorithms on the predicted results and the risk of clouding the conclusions relating to the use of these
algorithms by including other complex processes, no attempt to model the processes of consolidation of freshly deposited sediment was included in the modelling. Freshly deposited sediment was assumed to have the same properties as sediment not yet eroded from the bed. It is recognised that this factor will have an effect on the conclusions drawn here, particularly for 3D models. In particular, areas of deposition will result in fluid-mud layers which will act to further damp turbulence and enhance further deposition in these areas.

3.2 MODELLING APPROACH

The sediment transport simulations were undertaken using the TELEMAC-3D and SUBIEF-2D mud transport models from the TELEMAC suite developed by LNHE. These models are described in detail in Malcherek et al (1996), EDF (1994, 1997, 1998). For details regarding these models please contact HR Wallingford Ltd, UK, or SOGREAH, France.

The grid and bathymetry used for the simulations is shown in Figures 1 and 2. The model contained around 2000 nodes and, in the 3D model, between 5 and 10 (equally or differently spaced) layers making 10,000-20,000 nodes in all. The model domain is 3km along the shoreline with an inter-tidal mudflat around 700m from LW line to the sea wall.

The tidal range in the model was set at 3m with HW reaching 1m above the bed level at the HW coastline. The “seawall” boundary and the offshore boundary are no flow boundaries. The boundary condition for sediment entering the model was set at 100mg/l. Mud was assumed to be present everywhere on the bed within the model domain throughout the period of the simulation.

The waves were considered to have constant wave height, \( H_s = 0.4 \text{m} \), and period, \( T_p = 2 \text{s} \), throughout the model domain. The orbital wave velocity was calculated using the methodology of Soulsby and Smallman (1986) for irregular waves. To account for the effect of wave breaking the wave height wave height was limited to a height equal to 50% of the water depth.

Simulations were undertaken for a tide and 2½ hours. The effect of the wave was gradually increased during the first 2½ hours.

The fixed parameters used in the simulations are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical threshold for deposition</td>
<td>0.2 N/m²</td>
</tr>
<tr>
<td>Critical threshold for erosion</td>
<td>0.2 N/m²</td>
</tr>
<tr>
<td>Partheniades erosion rate constant</td>
<td>2.0x10^5 kg/m²/s</td>
</tr>
<tr>
<td>Settling velocity</td>
<td>0.5 mm/s</td>
</tr>
<tr>
<td>Density of in situ and deposited sediment</td>
<td>500 kg/m³</td>
</tr>
</tbody>
</table>

3.3 MODELLING OF ROUGH AND SMOOTH TURBULENT SCENARIOS

As noted by Soulsby (Soulsby and Clarke, 2004) rough turbulent flow occurs on coarser beds while smooth turbulent flow can be found to occur over a freshly deposited mud bed. In applied modelling of estuary systems, however, it is invariably the case that the tidal flows within the system as a whole are best reproduced by the rough turbulent model, even when the system can be described as “muddy”. It is rare that a model will solely be concerned with an area where a smooth-turbulent model is appropriate
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throughout. Applied estuary models are also generally constructed so that either a rough-turbulent or a smooth-turbulent model can be applied, but not both.

In the case where the interest lies in the modelling of a specific mudflat(s) some simplification is therefore usually required. Wave activity (whether smooth-turbulent or rough-turbulent) over a mudflat will generate levels of bed shear stress which are many times, even an order of magnitude, higher than the bed shear stress resulting from the small tidal currents that generally occur on mudflats. As the current-induced bed shear stress over mudflats is generally small, even for the rough-turbulent law, it is possible to use the rough-turbulent law to model currents with an estuary channel and over the mud flat. This simplifies the modelling greatly but does not compromise the calculation of the much larger wave erosion of the bed which can either be calculated on the basis of rough turbulent or smooth turbulent laws (the difference between these models can be described in one or two lines of program code) as appropriate. This approach will, however, over-estimate bed shear stress on the lower foreshore and channel where currents become sufficient to exert significant shear stress.

The modelling approach described below uses rough turbulent flows throughout and makes use of describing the wave action as smooth-turbulent or rough-turbulent as appropriate.

4. Results for smooth-turbulent results

4.1 BASELINE 2D RESULTS

The baseline results here represent depth-averaged mud transport using a constant settling velocity of 0.5mm/s and using Krone’s equation for deposition (Equation 1). The suspended sediment concentrations at times throughout the first tide along the profile of the intertidal are shown in Figure 3, together with the changes in bed level at the end of the simulation.

4.2 EFFECT OF WINTERWERP’S DEPOSITION ALGORITHM 1

The effect of removing the critical threshold of deposition on the predicted erosion is shown in Figure 3. Compared to the baseline case it can be seen that the resulting erosion of the bed is very similar on the upper profile but that on the lower profile the erosion is reduced. This results from the deposition on the bed that (according to Winterwerp) is occurring simultaneously with the erosion, with a consequently lower resulting net erosion.

The effect of removing the critical threshold of deposition on the predicted depth-averaged concentrations is shown in Figure 4. In order to keep the diffusion of sediment from the subtidal channel similar for both tests, the erosion from the subtidal channel more than 1000m from the HW line was kept the same. Between 600m and 100m is subtidal foreshore and over this distance the concentrations produced by wave activity for the scenario with deposition threshold increases as erosion continues. As the tide rises concentration reaches a peak where the combined shear stress due to waves and currents is at a peak. Following this point the currents reduce and the resulting erosion reduces. By comparison the scenario without a deposition threshold exhibits less erosion on the subtidal foreshore due to predicted deposition reducing the net erosion from the bed (Figure 3). As a result the predicted concentrations on the lower profile are reduced (Figure 4). Near High Water, therefore, during the slack
water when the erosion from both currents and waves is reduced there is less deposition on the upper profile and increased net erosion in comparison with the scenario with the deposition threshold.

For this particular example the difference in predicted net erosion rate is no more than 10% between these scenarios and the differences in predicted concentration is up to 20%. However, a much larger effect can result from more significant wave action (See Chapter 5).

From now on all of the modelling described below adopts the Winterwerp algorithm of no critical threshold for deposition.

4.3 EFFECT OF 2D/3D MODELLING – NUMERICAL APPLICATION 1

The effect of using a 3D model rather than a 2D model (compared to the 2D results) on predicted bed erosion and concentrations is shown in Figures 5 and 6. The figures show that there is a significant difference between the behaviour of the two models, as more sediment is eroded from the subtidal foreshore in the 3D model and a significant proportion of this sediment is deposited on the upper profile, reducing the extent of erosion. The 3D results show significantly higher concentrations (compared to the 2D results) which appear to be caused by enhanced erosion of the subtidal foreshore rather than the mudflat itself.

The observed difference can be attributed to following:

• The different friction laws utilised in the models – the 2D model here uses a Chezy friction law with a roughness value of 80m$^{1/3}$/s while the 3D modelling uses a Nikuradse roughness of 0.3mm. If you compare these friction factors in terms of the resulting bed shear stress, the 3D friction is higher as the Nikuradse law takes account of the shallow water depths of the intertidal area while the Chezy law doesn’t.

• The 2D model distributes flow in a horizontal spatial sense but the TELEMAC-3D model requires boundary condition at the bed. The type of boundary condition utilised by this model is a Neumann bed boundary condition (the derivative of the flow is specified at the bed rather than the value of the flow itself) which means that the calculation of current flow and resulting bed shear stress can be very different between the two models. This fact is most likely to contribute to the observed differences.

• To a small extent, some of the differences may be a result of the 3D nature of the flow.

Note, however, that even though the bed shear stress derived by the two models may be very different, the results still repeat the qualitative behaviour suggested in Section 4.2, that the greater the erosion rate of sediment on the subtidal foreshore, the greater the reduction in erosion on the mudflat above Low Water. Moreover, it is clear that the 3D model tends to reduce sediment on the upper profile at the expense of the lower profile and subtidal foreshore while the 2D model tends to produce erosion that is more evenly distributed or greater on the upper foreshore. There is thus a suggestion that the 3D model is transferring sediment from the lower foreshore to the upper profile while the 2D model is doing exactly the opposite.

Eroding foreshores are characterised by the classic “S” shape suggesting that real waves tend to produce erosion from the lower foreshore, moving material to the upper
foreshore, as predicted by the 3D results. However, it is necessary to compare the predictions of the model against real data to make conclusions regarding the accuracy of the predictions of erosion.

4.4 SOULSBY’S FORMULA FOR COMBINED BED SHEAR STRESS DUE TO WAVES AND CURRENTS – ALGORITHM 2

The effect of using the 3D-adapted version of Soulsby’s formula for combined bed shear stress due to waves and currents is compared to the effect of adding the current-induced and wave-induced bed shear stress together in Figure 7. The figure shows little difference in erosion on the upper profile of the mudflat but the additive wave model appears to show reduced erosion on the lower profile and subtidal foreshore. This is consistent with the Soulsby wave formulation which, in essence, produces a significant non-linear effect when the bed shear stress is of the same order of as the wave activity. The mudflat is a wave-dominated domain and so does not show an effect until the subtidal foreshore where currents are more substantial.

4.5 EFFECT OF 3D MODELLING AND INCLUSION OF TURBULENT DAMPING IN THE BED BOUNDARY CONDITION – ALGORITHM 3

The effect of 3D modelling and inclusion of turbulent damping in the bed boundary condition is shown in Figure 8. The results indicate the presence of what appears to be some minor instability near the Low Water Mark but otherwise show that the effect of turbulent damping is minimal. This is largely because the effect of waves in shallow water causes sediment to be very well mixed through the water column and there is almost no damping as a result.

4.6 MANNING’S ALGORITHM FOR SETTLING VELOCITY – ALGORITHM 4

The effect of using variable settling velocity is shown in Figures 9 and 10. The use of a the Manning predictive formula allows values of settling velocity to be up to 40% higher and 20% lower (i.e. 0.4-0.7m/s) than the chosen constant value of 0.5mm/s. The increased settling velocity results in more deposition on the bed and a reduction in erosion on the lower profile and subtidal foreshore. The implementation of the formula produces changes of up to 1mm/tide on the lower profile and subtidal foreshore, but limited change on the upper profile.

The effect shown here of including the Manning algorithm in the modelling appears relatively small to some extent. However, the importance of accurate prediction of settling velocity increases with increasing wave action (See Chapter 5).

4.7 EFFECT IN INCREASING VERTICAL RESOLUTION NEAR THE BED – NUMERICAL APPLICATION 2

The results of using variable settling velocity from Section 4.6 above derived using five equally spaced layers, were compared to simulations with ten evenly spaced layers and with ten layers spaced to give four times the resolution near the bed. The results are shown in Figure 11. The results show that the increased resolution reduces the amount of erosion on the lower intertidal profile and increases deposition on the upper profile. The reason for this is that the increased vertical resolution allows a better description of the near-bed gradient in density, leading to larger Richardson numbers, increased turbulent damping and lower bed shear stresses. These results are similar to those found
by Winterwerp (1999) using a 1DV model and investigating the onset of turbulent collapse in the sense that here increasing vertical resolution near the bed leads to enhanced deposition (and reduced erosion).

The conclusion is that the necessary discretization of the vertical profile into a limited number of layers will cause an over-estimation of erosion. In this case the range of predicted erosion resulting from changes in near bed vertical resolution is up to 2mm though this result is clearly a function of the wave height and larger waves may cause a larger range of results for varying levels of near-bed resolution.

4.8 EFFECT OF BIOLOGY – ALGORITHM 5

As described in Section 3.2, two different biological scenarios were selected to compare to the reference case. The first scenario was selected to be a scenario in which the biology produces an environment where the critical erosion threshold is relatively high and the rate of erosion is relatively low, i.e. a very depositional environment. The second scenario was selected to be representative of biology producing a reduction in the erosion threshold and an increase in the erosion rate.

The results of these two simulations are compared to the reference result in Figure 12. The figure shows the varying extent of erosion that occurs as a response to populations of deposition-enhancing and erosion-enhancing biology. The range in predicted erosion/deposition resulting from the biology is of the order of 0.5-2.5mm (per tide) on the intertidal area which is roughly of the same order as the effect of changes in vertical near-bed resolution (Section 4.7), but not as much as the differences that can occur between 2D and 3D model predictions (Section 4.3). Note that in this example the biology can change the mudflat from an erosional environment to a depositional environment.

5. Results for rough turbulent friction law

The results described above were for smooth turbulent mudflat conditions. Here we show the effect of representing rough turbulence in the 2D without a critical deposition threshold (i.e. with Algorithm 1 by Winterwerp), firstly with an arbitrary settling velocity of 0.5mm/s and secondly with variable settling velocity (Algorithm 4 by Manning). These results are compared to the 2D smooth turbulent result described in Section 4.2. Also presented are the 3D smooth turbulent and rough turbulent results incorporating all the algorithms (5 equal spaced layers, turbulent damping of bed shear stress, Soulsby’s method for combining wave and current bed shear stress and Manning’s algorithm for variable settling velocity).

The results are summarised in Figure 13. The results show the effects of increasing the wave activity, in this case by using a rough turbulent law, (but equally this could represent a smooth turbulent scenario for a much larger wave than the 0.4m used in this study).

For the 3D results, using the rough turbulent law increases erosion by up to 12mm (with a mean of 2-3mm) when compared to the 3D smooth turbulent wave result, except at the HW shoreline where erosion of up to 40mm is predicted. The result is less “smooth” indicating that the potential for numerical instability is greater for simulations involving more significant wave action.
For the 2D simulations using a rough turbulent law and a constant settling velocity of 0.5mm/s results in a twenty-fold increase in erosion. However, using the Manning algorithm reduces this increase in erosion to around twofold, or by around 2-5mm, in line with the rough turbulent 3D prediction. These results underline the importance both of allowing simultaneous deposition and erosion (Algorithm 1 by Winterwerp) and a good estimate of settling velocity (such as Algorithm 4 by Manning). Allowing simultaneous deposition and erosion means that for more significant wave action the extent of erosion can be severely reduced but if the estimate of the deposition flux, which relies heavily on settling velocity, is inaccurate then the resulting prediction of erosion can itself be wildly inaccurate.

6. Discussion and conclusions

The modelling studies described above exhibit results from a single mudflat with arbitrary conditions which be considered as “typical” but on no account covers the range of conditions for such systems. The results of the modelling are therefore qualitative and are intended to identify the important processes (and hence algorithms) for “typical” mudflat systems. Here, some of the more significant conclusions are summarised.

The implementation of Winterwerp’s algorithm for removing critical deposition threshold and instead calculating net change to the bed as the net result of erosion and deposition appeared to have a small effect under conditions of less significant wave activity. However, under conditions of more significant wave activity, the combination of the Winterwerp algorithm with the effect of allowing the settling velocity to vary (in this case using the Manning algorithm) produced very significant changes in the prediction of erosion of around an order of magnitude. It is important to note that without implementing the Winterwerp algorithm there is no deposition on the mudflat, except around High Water, regardless of the value of the settling velocity.

The choice of 2D or 3D model can also have a significant effect on the prediction of erosion. To some extent this result is model dependent and other modelling systems may be geared to provide more continuity between 2D and 3D. However, the modeller should in general expect large differences between 2D and 3D models owing to the different treatment of the bed boundary condition.

The Soulsby algorithm appears to have a small effect largely because of the nature of mudflats being a wave-dominated regime. In the case of moderate to high waves on intertidal areas, \( \tau_w + \tau_c \approx (\tau_w^2 + \tau_c^2)^{1/2} \approx \tau_{\text{max}} \) where \( \tau_{\text{max}} \) is the maximum bed shear stress predicted by Soulsby’s wave current interaction algorithm. The higher accuracy provided by the method, and the use of the mean value of \( u^* \) to decide the distribution of sediment and through the water column does not seem to have a significant effect in this situation. The algorithm does appear to be important near Low Water however where larger currents may occur.

One of the most significant effects on the predicted erosion on the mudflat is the near bed vertical resolution. This remains a significant problem to overcome in applied modelling as the model run time is nearly proportional in most cases to the number of layers.

It is clear that biology can cause very different effects depending on the nature of the species (bio-stabiliser or bioturbator). Note also that the range of sediment parameters
encountered in the field is greater than the range of sediment parameter values considered in this study due to the variety of different biological species and different sedimentary environments that may be encountered.

For this particular test case it appears that the inclusion of the Winterwerp algorithm involving simultaneous erosion and deposition in combination with a reliable estimate of settling velocity (such as the Manning algorithm) has the largest impact when modelling the erosion arising from more significant wave action. For smaller waves, where suspended sediment concentrations induced by wave action are lower, numerical considerations such as choice of 2D or 3D model and the number of layers included in the latter, appeared to be the most significant factors on the prediction of erosion on mudflats, although varying the sediment erosion parameters, as affected for instance by biology, also had a very significant impact on the resulting prediction of erosion.

7. Acknowledgements

I would like to acknowledge the kind assistance of my colleague and friend John Baugh whose expansive knowledge of the workings of the TELEMAC system came to my rescue many times.
8. **References**


Krone, R.B. (1962). Flume studies of the transport of sediment in estuarial shoaling processes, final report. Hydraulic Engineering and Sanitary Engineering Research Laboratory, University of California, Berkeley, USA.


Tables
## Table 1: Summary of simulations undertaken and algorithms/numerical applications used - Smooth Turbulent simulations

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Numerical applications</th>
<th>3D NEAR BED Vertical resolution</th>
<th>2D</th>
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<td>(variable settling velocity, Soulsby)</td>
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<td>4 (variable settling velocity, Manning)</td>
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<tr>
<td>Relevant Figures</td>
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Note on the use of algorithms for modelling mud transport on tidal flats.
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<th>Simulation</th>
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Appendix
Appendix 1  Extension of Soulsby and Clarke (2004) algorithm to 3D case

Soulsby and Clarke (2004) developed an explicit formula for shear stress due to combined wave and current interaction in terms of depth averaged currents. Here we repeat Soulsby’s analysis with minor modifications to extend his formula for 3D modelling purposes.

For a current in water of depth h, with density \( \rho \), current speed \( U_1 \) at height \( z_1 \) above the bed and bed roughness \( z_0 \), the bed shear stress \( \tau_c \) is given by:

\[
\tau_c = \rho C_{D1} U_1^2
\]

where

\[
C_{D1} = \left( \frac{\kappa}{\ln(\alpha z_1/z_0)} \right)^2
\]

and \( \alpha(z, \rho) \) is a correction factor accounting for the drag reduction caused by density gradients (Toorman, 2000). \( \alpha \) is related to the eddy viscosity damping function \( F_m \) by,

\[
F_m = \frac{1}{1 + \frac{z}{\alpha} \frac{\partial \alpha}{\partial z}}
\]

For a sinusoidal wave having period \( T \), and amplitude of the orbital velocity \( U_w \), the amplitude of the bed shear-stress \( \tau_w \) is given by:

\[
\tau_w = \frac{1}{2} \rho f_{wr} U_w^2
\]

where

\[
f_{wr} = 1.39 (A/z_0)^{-0.52} \quad \text{in the case of rough turbulent flow or,}
\]
\[
f_{wr} = 0.0521 \text{Re}_w^{-0.187} \quad \text{in the case of smooth turbulent flow}
\]

\[
A = U_w T/2\pi
\]

Most of the analytical and computational models of rough-bed (or smooth-bed) WCI make use of an eddy-viscosity assumption, which in general form can be written as:

\[
\tau = \rho \varepsilon \frac{\partial U}{\partial z}
\]

where \( \tau \) is shear-stress, \( \varepsilon \) is eddy-viscosity, and \( U \) is velocity at height \( z \). The effect of damping of turbulence by buoyancy means that the eddy viscosity is additionally a
function of the gradient Richardson number, itself a function of the gradients of current speed and density.

It is usual to consider the thin wave boundary layer (wbl), within which intense turbulence is generated in a layer a few centimetres thick above the bed, separately from the outer flow above this. We therefore specify a steady eddy-viscosity profile, inside and outside the wave boundary layer, with a matching current velocity at the interface. It is not strictly necessary for the eddy-viscosity to be continuous at the top of the wbl, and in many theories (e.g. Grant and Madsen, 1979) it is not. We make a valuable simplifying assumption, making use of the experimental finding by Simons et al (2000) that the oscillatory (wave) component of the stress is not enhanced by the current (at least for wave-dominated combinations). This allows us to specify the eddy-viscosity inside the wbl in terms of only the known quantities $\tau_c$ and $\tau_w$, which makes the problem greatly simpler than previous theories, and also explicit. Other theories usually express the eddy-viscosity as a function of the unknown quantities $\tau_m$ and $\tau_{max}$, which makes the equations implicit and requires an iterative solution.

Outside the wbl we assume:

$$\varepsilon = \kappa u_m F_m z (1-z/h) = \varepsilon_0 F_m$$  \hspace{1cm} (8)

where $u_m = (\tau_m/\rho)^{1/2}$, $\kappa = 0.40$ is von Karman’s constant, and $F_m$ is the damping function associated with the eddy viscosity.

This form ensures that the velocity and shear-stress profiles in this region (i.e. through most of the depth) are consistent with a mean bed shear-stress of $\tau_m$. Inside the wbl we assume

$$\varepsilon = \kappa u_e F_m z = \varepsilon_0 F_m$$  \hspace{1cm} (9)

where

$$u_e = (\tau_e/\rho)^{1/2}$$  \hspace{1cm} (10)

and

$$\tau_e^2 = \tau_c^2 + \tau_w^2$$  \hspace{1cm} (11)

Equation (11) is at the heart of the Soulsby method. It states that the “effective stress” $\tau_e$ for the purpose of setting the velocity-scale inside the wbl is given (explicitly) by the root-mean-square of the separate current and wave shear stresses.

The thickness $\delta$ of the wbl is assumed to depend only on the oscillatory part of the velocity, and, furthermore, to depend on the wave-alone friction velocity $u_{*w}$, as:

$$\delta = a_r \frac{u_{*w}}{\omega}$$  \hspace{1cm} (12)

where $a_r$ is a constant to be determined empirically, and $\omega = 2\pi/T$. Similar expressions have been used in previous theories (e.g. Grant and Madsen, 1979), but with the friction velocity including an element of the steady-current shear-stress. The form given by Eq. (12) is preferable because (a) the physical process of formation of the wbl is essentially oscillatory, (b) use of the known quantity $u_{*w}$ makes Eq. (12) explicit.
Inside the wbl \((0 \leq z \leq \delta)\), assuming \(\delta \ll h\) and hence \(\tau(z) \sim \rho u_{m}^2\), Eq. (7) becomes:

\[
\rho e_{0} F_{m} \frac{dU}{dz} = \rho u_{m}^2
\]  

(13)

and using Eq. (4),

\[
\kappa u_{e} \frac{dU}{dz} = u_{m}^2
\]  

(14)

Integrating Eq. (14) w.r.t. \(z\), and applying the boundary condition \(U(z_{0}) = 0\), gives the velocity profile inside the wbl:

\[
\int_{0}^{u} f_{m} du = \frac{u_{m}^2}{\kappa u_{e}} \ln \left( \frac{z}{z_{0}} \right)
\]  

(15)

where \(f_{m}\) is given by (Toorman 2000),

\[
f_{m} = F_{m} \left[ 1 - \frac{\ln \alpha}{\ln(z/z_{0})} \right]
\]  

(16)

Outside the wbl \((\delta \leq z \leq h)\), the shear-stress decreases linearly with height (this follows from time-averaging and vertically-integrating the equation of motion):

\[
\tau(z) = \tau_{m} \left( 1 - \frac{z}{h} \right)
\]  

(17)

Using Eq. (8) and (9) in (17) gives:

\[
\kappa u_{m} \left[ 1 - \frac{z}{h} \right] \frac{dU}{dz} = u_{m}^2 \left( 1 - \frac{z}{h} \right)
\]  

(18)

Integrating Eq. (18) w.r.t. \(z\), and applying a matching condition at \(z = \delta\) gives:

\[
\int_{0}^{u} f_{m} du = \int_{0}^{\delta} f_{m} du + \frac{u_{m}^2}{\kappa} \ln \left( \frac{z}{\delta} \right)
\]  

(19)

Making use of Eq. (17) for \(U(\delta)\) gives the velocity profile outside the wbl:

\[
\int_{0}^{u} f_{m} du = \frac{u_{m}^2}{\kappa u_{e}} \ln \left( \frac{\delta}{z_{0}} \right) + \frac{u_{m}^2}{\kappa} \ln \left( \frac{z}{\delta} \right)
\]  

(20)

Equation (20) is a quadratic equation in the unknown \(u_{m}\), which has the positive solution:
\[ u_{*m} = \frac{1}{2A} \left[ B^2 + 4A \int_0^1 f_m du \right]^{1/2} - B \]  

(21)

where

\[ A = \frac{1}{\kappa w_c} \ln \left( \frac{\delta}{Z_o} \right) \]  

(22)

\[ B = \frac{1}{\kappa} \ln \left( \frac{z}{\delta} \right) \]  

(23)

If we consider a 3D model, the shear stress is computed at the bottom-most layer, \( z_1 \), according to Equations 1 and 2. In this case we can approximate the integral \( \int_0^1 f_m du \) as,

\[ \int_0^1 f_m du = U_1 f_{m1} \]  

(24)

where \( f_{m1} \) is calculated from the gradients in current speed and density between layers 1 and 2.

Equation 21 therefore becomes,

\[ u_{*m} = \frac{1}{2A} \left[ B^2 + 4AU_1 f_{m1} \right]^{1/2} - B \]  

(25)

where \( f_{m1} \) can be derived using Equation 16 and the following empirical approximation (Toorman 2000),

\[ \alpha = \exp \left[ - \left\{ 1 + 7.7 \frac{\tau}{u_*} \right\} \left\{ 1 - \exp \left( - \frac{R_{i,0.85}}{0.6} \right) \right\} \right] \]  

(26)

Both A and B contain only known quantities, so that Eq. (25) is an explicit expression for the mean friction velocity \( u_{*m} \), and hence the mean bed shear-stress \( \tau_m = \rho u_{*m}^2 \), under combined waves and currents as required. It can alternatively be written in terms of the mean drag coefficient \( C_{Dm} = \tau_m / \rho \bar{U}^2 \) as:

\[ C_{Dm} = \left[ \left( A_1^2 + A_2 \right)^{1/2} - A_1 \right]^2 \]  

(27)

where
The maximum shear-stress $\tau_{\text{max}}$ is derived by first considering the periodic zero-mean bed shear-stress corresponding to the wave-induced oscillations of the bed shear-stress about the mean value $\tau_m$. A linearising assumption is made such that the periodic stress is written as $\tau_p \sin(\omega t)$. Soulsby (1983, p208) solved this problem of a linearised shear-stress in an oscillatory boundary layer with a steady eddy viscosity increasing linearly with $z$. The result (Soulsby, 1983, Eq. 35) can be written in terms of the oscillatory wbl in the present problem, within the wbl ($z_o \leq z \leq \delta$) as:

$$\tau_p = \rho u_e U_w \left(\frac{f_{wr}}{2}\right)^{1/2}$$

(30)

For strict compatibility the wave friction factor $f_w$ should be obtained from the solution of the oscillatory equation given, for example, by Soulsby (1983, Eq. 36). However, this is a complicated expression in terms of the real and imaginary Kelvin functions ($\text{ker}$ and $\text{kei}$), which would be computationally intensive to calculate. Instead, in order to retain the same level of complexity as in the rest of the derivation, we choose to specify $f_w$ in the simple empirical form of Eq. (7). Then a vector addition of $\tau_m$ and $\tau_p$, for waves propagating at an angle of $\phi$ to the current direction gives:

$$\tau_{\text{max}} = \left[\left(\tau_m + \tau_p \cos\phi\right)^2 + \left(\tau_p \sin\phi\right)^2\right]^{1/2}$$

(31)

This expression can alternatively be written in terms of the maximum drag coefficient ($C_{D_{\text{max}}} = \tau_{\text{max}} / \rho \bar{U}^2$) as:

$$C_{D_{\text{max}}} = \left[ C_{D_m} + \frac{u_e}{U_{1,fr_{ml}}} \frac{U_w}{U_{1,fr_{ml}}} \left(\frac{f_{wr}}{2}\right)^{1/2} \cos\phi \right] + \left[ \frac{u_e}{U_{1,fr_{ml}}} \frac{U_w}{U_{1,fr_{ml}}} \left(\frac{f_{wr}}{2}\right)^{1/2} \sin\phi \right]^2$$

(32)

where $C_{D_m}$ is calculated from Eq. (27) and all the other quantities are known. The root-mean-square shear-stress is given by:

$$\tau_{\text{rms}} = \left(\tau_m^2 + \frac{1}{2} \tau_p^2\right)^{1/2}$$

(33)

References


