The relative importance of waves in predicting the morphology of UK estuaries

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THE RELATIVE IMPORTANCE OF WAVES IN PREDICTING THE MORPHOLOGY OF UK ESTUARIES

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Abstract
Recent developments provide a means of defining the estuary form based on external or pre-defined parameters such as tidal range, river discharge, valley length, nature of the valley bed and the type of sediments available as a supply. This was found to provide a reasonable representation of UK estuaries for properties such as surface area and volume at mean tide level but a much poorer representation of areas and volumes of the low water channel and the tidal flats. The inclusion of an idealised representation of the equilibrium profile generated by wave action was found to significantly improve the predictive power of the model when compared to the properties of 64 UK estuaries.

Introduction
Our ability to understand how the coastal zone is likely to evolve under climate change and use this understanding to devise suitable adaptation strategies depends on being able to integrate information across a range of spatial and temporal scales (Townend, 2004). Often the problem can be usefully constrained by the geology, knowledge of the Holocene evolution and information on the forcing conditions, such as sea level rise, tidal conditions and storminess. Whilst detailed field work and modelling of the processes provides much useful information it does not necessarily provide the broader view needed to establish a sufficient understanding on which to make management decisions. For this reason, simple models that seek to capture the behaviour and evolutionary trends of a system can provide a useful adjunct to more detailed process modelling.

Equilibrium methods have been extensively explored as a basis for describing some key properties of estuaries. The best known of these is the so called O'Brien relationship between tidal prism and cross-sectional area (O'Brien, 1931). More recent endeavours have sought to provide a physically based explanation of this and similar relationships (Friedrichs, 1995; Kraus, 1998; Hughes, 2002). These are all predominantly based on tidal flow although there is often some acknowledgement that the effect of littoral transport (and hence waves) can influence the scaling of this relationship at the mouth of an estuary.

However, for many applications it would be useful to know about other system properties, such as the area and volume of the channel and tidal flats. An investigation of UK estuaries suggested that these properties were also strongly correlated to variable such as the tidal prism (Townend, 2005). Here we seek to give a more systematic basis for such form relationships, by considering equilibrium arguments to determine the plan form, and
cross-sectional form of the estuary. Combining the two then gives a 3D model of the estuary bathymetry, from which the gross properties of volume and surface area for the channel and tidal flats can readily be derived.

This type of model serves a number of purposes. It provides an alternative basis for prediction and can be used as a “reality” check for the results from more detailed process based models. The understanding derived from this approach also allows the further development of long-term morphological models to be given the appropriate focus. In terms of applications, the ability to predict, a priori, what the form of a system is likely to be knowing only the external, or pre-defined, conditions is critical to the prediction of how habitat creation and managed realignment schemes are likely to evolve. The results presented here provide some progress towards this objective.

Outline of Model

As noted in the introduction, a number of authors have sought to define a theoretical basis for the observed relationship between tidal prism and cross-sectional area (Spearman, 2007). This typically considers some critical erosion shear stress for sediment movement and derives the equilibrium scour depth or area based on some characteristic flow equation (e.g., Manning–Stickler). The discharge is then related to the tidal prism by assuming a time average discharge and noting that $Q \sim P \pi / T$ where, $Q$ is discharge, $P$ is the tidal prism and $T$ is the tidal period.

An alternative approach is to examine the characteristic dimensions of estuaries using the hydrodynamic equations and making some simplifying assumption, such as assuming a simple prismatic cross-section (Prandle, 2006). An analytical approach to estuarine morphological predictive modelling, referred to as the Analytical Emulator (AE), was originally developed by Prandle (2003). The AE is largely based on the one-dimensional equations of axial momentum and continuity. A number of general rule-based morphological explicit expressions were derived by Prandle (2004) which included a description of estuarine depth in terms of the river flow and channel side slope. The AE has been successfully implemented by Manning (2008b).

Table 1 – Characteristic properties of estuaries

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landscape setting</td>
<td></td>
</tr>
<tr>
<td>Basin area at high water (for inlets)</td>
<td>$S_{hw}$</td>
</tr>
<tr>
<td>Length of the estuary (~ distance from mouth to point in valley where high water intersects the valley slope), ie the tidal limit</td>
<td>$L_e$</td>
</tr>
<tr>
<td>Tides</td>
<td></td>
</tr>
<tr>
<td>Tidal amplitude at mouth</td>
<td>$a$</td>
</tr>
<tr>
<td>Tidal period</td>
<td>$T$</td>
</tr>
<tr>
<td>River flow</td>
<td></td>
</tr>
<tr>
<td>River discharge</td>
<td>$Q_r$</td>
</tr>
<tr>
<td>Winds</td>
<td></td>
</tr>
<tr>
<td>Dominant wind speed</td>
<td>$U_w$</td>
</tr>
<tr>
<td>Sediments/geology</td>
<td></td>
</tr>
<tr>
<td>Sediment grain size</td>
<td>$D_{50}$</td>
</tr>
<tr>
<td>Bulk density of bed sediments</td>
<td>$\rho_b$</td>
</tr>
<tr>
<td>Critical bed shear stress</td>
<td>$\tau_{cr}$</td>
</tr>
<tr>
<td>Concentration of suspended sediment supply</td>
<td>$c_n$</td>
</tr>
</tbody>
</table>
Here we make use of a number of well established relationship to derive an idealised 3D form of the estuary (discriminating between channel and intertidal) based on characteristic properties of the estuary setting, such as the size of the drainage basin, and properties that are determined outwith the estuary, such as tides and river discharge. The properties used to determine the form are summarised in Table 1. There is some debate as to whether sediments should be regarded as a governing property or are themselves determined by the hydraulics (Friedrichs, 2006; Prandle, 2004). Here we take the view that in an eroding system the bed properties are determined by the antecedent geology and where deposition takes place this is necessarily dependent on the characteristics of the available supply – marine or fluvial – sands or silts.

A number of components are combined to provide the overall model. The estuary is represented as an exponentially converging system, with cross-sections throughout that combine the equilibrium form of the channel and tidal flats. In order to prescribe this form based on only external or antecedent parameters, it is necessary to make use of a number of hydraulic and sediment transport relationships. At the tidal limit, one must either know the dimensions of the river channel, or derive the dimensions using a suitable regime theory (eg Cao & Knight, 1996). The estuary is dimensioned by equating the hydraulic properties of tidal propagation (Friedrichs & Aubrey, 1994) to the geometric properties of the tidal exchange, making use of the limiting conditions for erosion to obtain the critical threshold velocity, as modified by deposition due to the amount of sediment in suspension. These parameters are then sufficient to allow the form model to be dimensioned and the gross properties (areas and volumes of the channel and tidal flats) of the system to be obtained.

A further refinement is to introduce the additional influence of waves on the cross-sectional form. For estuaries that are macro-tidal, the tidal form will dominate and the influence of locally generated waves will be secondary (eg Humber estuary). However for systems with smaller tidal ranges in exposed locations, waves can be considerably more important (eg Poole Harbour).

**Plan form**

The along estuary variation in cross-sectional area, $A$, at mean tide level (mtl) is defined by:

$$A(x) = A_m \cdot \exp \left( -\frac{x}{L_A} \right)$$  \hspace{1cm} (1)

Where $A_m$ is the area at the mouth of the estuary, $x$ is the distance along estuary from the mouth and $L_A$ is the e-folding length for cross-sectional area, as distinct from the length of the estuary, $L_e$. By assuming that the hydraulic depth at mean tide level, $h_0$, remains constant, then $L_A = L_W$ and a similar expression describes the along estuary variation in width.

The e-folding length is determined from the hydraulics of the system by noting that for convergent systems, with the tidal frequency denoted by $\omega$, the amplitude of the tidal velocity, $U$, is given by:

$$U = \frac{a \cdot \omega \cdot L_A}{h_0}$$  \hspace{1cm} (2)

In addition, using well known relationships for the rate of erosion and deposition (Whitehouse *et al.* 2000) we obtain the bed shear stress at equilibrium, $\tau_0$, as a function of the erosion shear stress, $\tau_{cr}$, long-term average concentration, $c_n$, and representative sediment fall velocity, $w_s$ by equating the erosion and the deposition and taking the time-average over a representative time scale:

$$\frac{dm}{dt} = m_e \left( \tau_0 - \tau_{cr} \right) \text{ for } \tau_0 > \tau_{cr}$$  \hspace{1cm} (3)

and 

$$\frac{dm}{dt} = -w_s \cdot c_n$$
Equation (3) provides an alternative estimate of the tidal velocity, $U$ as a function of depth, $h_0$. We therefore have three unknowns, $U$, $h_0$ and $L_ε$ and so need a third equation to find a solution. This is provided by integrating the equation for tidal discharge to obtain an estimate of the prism, assuming an exponentially convergent system and hence an approximate estimate of the e-folding length, $L_W$:

$$L_W = \frac{1}{k} \cdot \tan^{-1}\left(\frac{2ε_H}{1-ε_H^2}\right)$$  \hspace{1cm} (4)

where $k$ is the tidal wave number, given approximately by $k = \frac{ω}{\sqrt{gh_0}}$ and $ε_H = \frac{πa}{4h}$ (where $h$ is the hydraulic depth over the channel width). Equations (2)-(4) can now be solved to find the hydraulic depth, $h_0$, and whence the velocity and width e-folding length.

**Channel and tidal flat**

A close correspondence between the equilibrium form relationships for both estuaries and rivers suggests that “estuaries do not substantially differ from alluvial rivers in terms of morphology” (Savenije, 2005). Given that the focus in this paper is on gross properties, without regard for detailed features such as flood and ebb channels, such a simple description should suffice. The combined form of low water channel and intertidal flats is illustrated in Figure 1, which also defines some of the key dimensions. For the low water channel we make use of an approach originally derived for river channels (Cao & Knight, 1996).

The equilibrium form of the intertidal profile has been examined using arguments based on energetics in relation to wave activity (Dean, 1991; Lee & Mehta, 1995) and tidal propagation or the combination of the two (Friedrichs & Aubrey, 1996). Alternative approaches include the use of detailed process models (Roberts et al. 2000; Woolnough et al. 1995), or consideration of the basin hypsometry, which has been used to examine the overall form of creeks and estuaries (Boon III & Byrne, 1981; Wang et al. 2002; Townend, 2008). To provide a simple description of the intertidal profile, the concept of uniform dissipation of energy is used, with the square of the maximum depth averaged velocity as a proxy for the maximum bottom shear stress (Friedrichs & Aubrey, 1996).

Combining the plan form model for the variation in width along the estuary, with the channel and intertidal cross-shore profiles, provides a basis for generating an idealised bathymetry of the estuary. The
resulting 3D form model allows a number of integral properties to be derived as measures of the overall size of the estuary. For instance, simply integrating equation (1) provides the volume and integrating a similar equation for width gives the surface area at mean tide level:

\[ V_0 = A_{ml} \cdot L_A \left[ 1 - \exp\left( \frac{-L_c}{L_A} \right) \right] \]
and
\[ S_0 = W_{ml} \cdot L_W \left[ 1 - \exp\left( \frac{-L_c}{L_W} \right) \right] \]  

By integrating the equations for the profiles of the tidal flat and channel, over the intervals to which they apply, cross-sectional areas are obtained. Knowing the widths and areas of the profile leads to the following simple expressions for surface areas, \( S \), and volumes, \( V \), of the channel and intertidal flats:

\[ S_{ml} = S_0 - n_{bk} \cdot a \cdot m_{eqS} \quad V_{ml} = V_a - \frac{a}{2} (S_{in} + S_0) \]

\[ S_{fl} = \left( \frac{1 + \pi}{2} \right) \cdot n_{bk} \cdot a \cdot m_{eqS} \quad V_{fl} = \frac{1 + \pi}{2} n_{bk} \cdot a^2 \cdot m_{eqS} \]

(6)

Where \( n_{bk} \) is the number of banks, \( m_{eqS} \) is the slope of the lower intertidal (estimated from the rate of convergence and tidal velocity, \( U \). Suffix 0 relates to mean tide level values, \( lw \) to the channel values and \( fl \) to the values for the tidal flat.

**Modification of profiles by waves**

The role of waves is known to vary along the length of an estuary (Pethick, 1994). Towards the mouth, waves propagating from offshore can be a dominant influence shaping both the intertidal and the shoals and spits that develop due to the interaction of littoral processes on the open coast and the estuary. Further upstream, waves are limited to those that are internally generated and as a consequence are generally fetch limited. To provide a first assessment of waves on the gross properties of estuaries the 3D form model described above is extended to include the influence of internally generated wind-waves (i.e., those that form and propagate along fetches within the estuary).

The wave field itself is based on a wind speed, \( U_w \), blowing over a fetch \( F \), using some form of parametric model. In this case the TMA spectrum (Bouws et al. 1985; Hughes, 1984) has been used because this includes the influence of depth limitation on the wave spectrum.

A similar argument to that used for tidal flows can be used to determine the critical depth for given wave conditions. If we again assume a sinusoidal wave form, then the velocity in excess of the erosion threshold, averaged over the wave period, can be obtained using the balance of erosion and deposition, equation (3). By using linear wave theory an explicit expression can be obtained for the equilibrium depth, \( d_e \), at which waves will begin to influence the profile.

The form of the wave generated profile is obtained by considering the conservation of energy across the profile (Dean, 1991; Friedrichs & Aubrey, 1996) to obtain

\[ d = d_e \left( 1 - \frac{V}{L} \right)^{2/3} \]  

(7)

Given the depth for the initiation of wave influence on the channel profile, the length of the profile, \( L \), is given by (Friedrichs & Aubrey, 1996):

\[ L = \frac{3\pi}{4c_d} \frac{d_e^2}{H} \]  

(8)

In equation (8) \( c_d \) is the friction coefficient and \( H \) the height of the dominant wave. For the model presented here we make the assumption that whilst waves will influence
the profile at all stages of the tidal cycle, the predominant influence on the form will occur during the high and low water stands, when the extended time at a fixed elevation increases the probability that waves will be able to leave a signature that tends towards the equilibrium profile. At high and low water estimates are made of the fetch based on the width of the estuary, and with the average depth of the estuary and the prevailing wind speed the wave height and period using the TMA spectrum, as noted above. Modifying the concentration to take account of water depth changes, equation (3) provides the wave profile depths at high and low water, from which the width of the two profiles follow from equation (8) and the elevations across the profile can be determined using equation (7). As these two profiles can now be defined relative to known water levels (high and low water) and points on the flow-only-profile where the depth equals the equilibrium depth for the wave profiles, the overall cross-section can be modified to include the wave profiles, Figure 2.

The wave profile in Figure 2 has a more concave upwards shape in the two zones of influence which are in accord with the erosional profile assumption of Kirby and Dyer, and the current only profile is convex upwards in accord with the accretionary profile assumption (Dyer, 1998).

**Estuary Data**

UK estuaries were first systematically catalogued by the Nature Conservancy Council (Davidson et al. 1991). This provided a template on which to develop a more extensive database of estuary properties. As part of the first phase of the Defra/EA Estuaries Research Programme, some gross properties were added based on an analysis of available chart data for a large proportion of the estuaries. In addition, more detailed data were collated for six estuaries. These data are available on the EMPHASYS database CD now held by BODC (EMPHASYS Consortium, 2000). Subsequent analysis assessed the accuracy and limitations of the gross properties data and explored some of the relationships already noted (Townend, 2005). The database was further extended as part of the Defra/EA Futurecoast project. This revisited the classification of estuaries and added a number of variables and began to compile some information on fluvial flows (Futurecoast Consortium, 2002). More recently the database has been further extended (Prandle et al. 2006).
As part of the “Data requirements – Objective 4” for the Defra funded project FD2107, the original JNCC/NCC (data for 155 UK estuaries) database and Future-Coast (data for 96 English & Welsh estuaries) database were combined and expanded to form the newly enhanced “ExpFC” database (Manning, 2008a). For English and Welsh estuaries, the following additions were made to the ExpFC database:

- Detailed freshwater flows (seasonal statistics) for 65 estuaries (source: Centre for Ecology & Hydrology archives).
- Saline intrusion lengths for most estuaries (source: literature reviews and Marine Nature Conservancy Review).
- Spring and Neap tidal ranges for all England and Wales estuaries (source: Admiralty Tide Tables).
- The following parameters were calculated: Tidal amplitudes; Mean, high and low water estuary depths; Mean estuary breadth and side-slope.

Also dimensional data from 110 Scottish sea lochs was digitised from a report compiled by Edwards and Sharples (Edwards & Sharples, 1986) and included as a supplement to the ExpFC database (SLscot ). Both the ExpFC and SLscot are jointly archived at the British Oceanographic Data Centre (BODC), Liverpool. Extracts from the database can also be found on www.estuary-guide.net.

Results

One of the notable shortcomings of the estuaries database is the lack of information on sediments. This meant that a full set of parameters was not available for most estuaries, so that the model could not be used to predict the gross properties based solely on the pre-defined parameters. In order to test the model, guestimates of the sediment parameters were made based on what is know about each estuary (sandy, muddy, or mixed). These initial values of grain size, bed shear stress and bed density were then adjusted to obtain the correct tidal prism, when wave influences are included, whilst being constrained to “reasonable” values, given what is known about the sedimentological character of each estuary. All the other gross properties were then compared to the observed value to evaluate how well the idealised form is able to represent real estuaries.

For brevity, only the results for volume and surface area at mean tide level are presented, Figure 3. Each plot compares the observed values with those predicted by the model. The dotted lines, define the 1:1 agreement and the dashed lines represent the power law regression to the data (scale and shape factors of 1 would indicate a good quality fit and the $R^2$ value reflects the scatter of the data about the regression line).

The left hand plots of Figure 3 show the results when only tidal flow is considered. For both area and volume the underlying trend is represented, although more so for area than volume. However there is considerable scatter in the results. Given that these are log-log plots, this scatter represents orders of magnitude error, in some cases, rather than some small percentage differences.

The right hand plots of Figure 3 provide a direct comparison with the inclusion of waves in the model. This addition greatly improves the estimates of surface area, which is now well represented, but there remains some scatter in the estimates for volume at mean tide level. From an examination of the other gross properties it is clear that this is largely due to the scatter in the estimates of low water channel volume.

Overall the surface area estimates for channel and flats are reasonable. The shape, scale and regression values for the channel are similar to the mean tide level values (see Figure 3). For the tidal flats they are respectively 9, 0.85 and 0.79 for surface area and 10.6, 0.88 and 0.83 for volume. In all cases the estimates of surface area are significantly improved by
the inclusion of waves, as is the volume of the tidal flats. In contrast, the channel volume shows only a minor improvement, suggesting that the error is dominated by the flow conditions and waves have only a minor influence.

Discussion

From an examination of those estuaries that are under or over predicted in terms of the channel volume at low water, it is clear that a primary reason is likely to be the specified flow conditions. For this study the mean annual river flow has been used. For those systems that are under-predicted, these values are often very small (less than 1m/s). It may be that high or spate flows are more significant in determining the morphology of the channel, and the distribution of sediment in the system (e.g. in the Thames and the Humber). Of those systems that are over predicted and have relatively high flows, it is probable that the geology is in some way constraining the system. However, many of these systems are sandy with a significant littoral supply, where bed load transport may supplement the import of suspended sediments.

Theoretical arguments have been presented that the overall dimensions of estuary systems are determined by the hydrodynamics, independent of the sedimentological properties (Friedrichs, 2006; Prandle, 2004). In the approach adopted for this analysis, shear strength, bed friction and suspended sediment supply all influence the determination of the form characteristics, namely tidal velocity, hydraulic depth and width e-folding length, as well as the tidal and fluvial conditions. Thus the overall dimensions and the way in which waves modify the intertidal form are both conditioned by the sedimentary properties of the system.

Conclusion

A simple equilibrium model is presented that extends the well known form of inlet relationship between cross-sectional area and tidal prism. This makes use of an idealised representation of the estuary, based on the plan form combined with the channel and intertidal cross-shore profiles. This provides a reasonable representation of gross properties of UK estuaries when only flow is considered but is much improved by the inclusion of the influence of waves.

The premise for the model was that it should be determined by exogenic properties, ie those that are defined by the environmental setting of the estuary. These are essentially the drainage basin setting (determines length or inlet area), external tides, river flows and sediment properties, either of the antecedent bed lithology (where the bed is eroding), or the sources of supply (for deposition). Although there is sufficient data available to examine gross properties, even these data are known to be of limited accuracy (Townend, 2005). It has also been necessary to estimate the sedimentological parameters and as noted in the discussion characterisation of river discharge data (discriminating spate and mean flows, etc) may need to be improved. In order to improve the estimates that can be made with this type of model and advance our understanding of equilibrium concepts within estuaries the following data needs to be captured or improved:

- Bathymetry (with particular emphasis on the intertidal and including a definition of any landward constraints);
- River flows (including determination of the influence of spate flows as against mean and annual peak flows – several systems undergo major change to sustained and/or very high flows that occur infrequently and they do not necessarily recover between events);
- Sediment properties (sediment grain size and bed density are a minimum requirement. Information on the threshold shear stress on the tidal flats and in the channel, as well as suspended sediment concentrations would help validate the relevant empirical relationships).

If this data were to be collected systematically for a number of UK estuaries
(ideally all) but, as a minimum, a set of 30 (for statistical purposes) that draws on the different types of estuary present (coastal plain, ria, etc) it would be possible to further develop some of the ideas presented here.
Figure 3 – Plots of Volume and Surface area at Mean Tide Level for flow only (left) and including waves (right)
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