SEDIMENT TRANSPORT MODELS FOR ESTUARIES
G V Miles

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Hydraulics Research Station
Wallingford
Oxon OX10 8BA
Telephone 0491 35381
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- \( q \): speed of current, \((\overline{u}^2 + \overline{v}^2)^{1/2}\) (ms\(^{-1}\))
- \( Q \): discharge per unit width (m\(^3\) s\(^{-1}\))
- \( R \): settling velocity Reynolds Number, \(w_d/2D_a\)
- \( R_b \): hydraulic radius
- \( s \): natural co-ordinate in direction of flow
- \( S \): source/sink term at the bed (kg m\(^{-3}\) s\(^{-1}\))
- \( S_d \): deposition rate (kg m\(^{-2}\) s\(^{-1}\))
- \( S_e \): erosion rate (kg m\(^{-2}\) s\(^{-1}\))
- \( S_p \): vertical flux of sand (kg m\(^{-2}\) s\(^{-1}\))
- \( t \): time
- \( T_s \): sand transport (kg m\(^{-2}\) s\(^{-1}\))
- \((u,v,w)\): components of velocity vector (m s\(^{-1}\))
- \((G, \nabla)\): depth integrated velocity components (m s\(^{-1}\))
- \( u_{\text{crit}} \): threshold velocity for sand transport (m s\(^{-1}\))
- \( v_o \): uniform velocity (m s\(^{-1}\))
- \( w_o(D) \): settling velocity (m s\(^{-1}\))
- \((x,y,z)\): cartesian co-ordinates
- \( z_b \): elevation of bed (m)
- \( a \): profile parameter, \(f(1/d-\overline{w})\)
- \( b(r) \): bed exchange factor
- \( \delta_k \): profile parameter, \(c(x,y,0)\overline{c}(x,y,0)\)
- \( \kappa \): Von Karman constant
- \( \nu_f \): eddy viscosity (m\(^2\) s\(^{-1}\))
- \( \rho_b \): consolidation ratio of deposited mud (kg m\(^{-3}\))
- \( \sigma \): dimensionless variable, \((w_d^3/4D_a)^2 \overline{v} \overline{u} \overline{w}\)
- \( \tau \): dimensionless variable, \((\sigma^a+D_a)^{1/3} \overline{v}^{1/3}\)
- \( \tau_b \): bed stress (Nm\(^{-1}\))
- \( \tau_d \): critical stress for deposition (Nm\(^{-1}\))
- \( \tau_e \): critical stress for erosion (Nm\(^{-1}\))
- \( \delta \): dimensionless vertical co-ordinate, \(z/d\)
SUMMARY

This paper is a review of numerical models for studying sediment transport with special emphasis on the applicability of methods to estuarine conditions. The main features of estuaries are the wide range of sediments present, the absence of erodible material in some places, a combination of unsteady and nonuniform flow and lateral as well as longitudinal variations in the flow and suspended solids concentrations. Saline stratification can also be present. Not surprisingly, no single model has so far been presented to simulate all the estuarine sediment processes but many models are described which can be used to study certain aspects. The models are separated into potential load models primarily geared to studying bed load transport, suspended sand models and suspended mud models.

The review covers the most significant papers published in recent years, supplemented by some case studies of projects carried out in the Hydraulics Research Station, Wallingford.

9 LIST OF SYMBOLS

- \( c(x,y,t) \) concentration of suspended solids (kg m\(^{-3}\))
- \( \overline{c}(x,y,t) \) depth-integrated concentration (kg m\(^{-2}\))
- \( c_0 \) initial concentration (kg m\(^{-3}\))
- \( \phi \) saturation ratio, \( \overline{c}/c_0 \)
- \( c_s \) concentration under saturated conditions
- \( c_s^* \) depth-integrated, saturated concentration
- \( \phi \) Chezy friction factor
- \( d \) water depth (m)
- \( D \) sediment grain size (mm)
- \( D_c \) lateral (turbulent) diffusion coefficient (m\(^2\) s\(^{-1}\))
- \( D_s \) longitudinal (shear) dispersion coefficient (m\(^2\) s\(^{-1}\))
- \( D_{sx}, D_{sy}, D_{sz} \) diffusion coefficients in 3-D axes (m\(^2\) s\(^{-1}\))
- \( E \) sand transport coefficient (kg m\(^{-2}\) s\(^{-1}\))
- \( F_z \) net vertical flux of sand (kg m\(^{-2}\) s\(^{-1}\))
- \( g \) acceleration of gravity (m s\(^{-2}\))
- \( m \) quantity of mobile bed material (kg m\(^{-2}\))
- \( M \) erosion rate (kg m\(^{-2}\) s\(^{-1}\))
- \( n \) natural co-ordinate normal to flow
- \( P \) probability of deposition
1 INTRODUCTION

An estuary is a partly enclosed body of tidal water where river water is mixed with and diluted by sea water. In a general sense the estuarine environment is defined by salinity boundaries rather than by geographical ones, but although the salinity has influence on the clay sediment fractions it is the currents generated by the tidal volume flowing in and out of the estuary which dominate the movement and distribution of sediments. The sediments themselves may have originated from natural erosion inland or from seawards. They consist of materials ranging from the finest clay particles to coarse sand and gravels. A convenient classification of sediments uses a geometric scale of sizes.

\[
\begin{align*}
\text{mm} & & \text{phi units} \\
\text{Very coarse sand} & & 1.0 - 2.0 & & 1 \\
\text{Coarse sand} & & 0.5 - 1.0 & & 2 \\
\text{Medium sand} & & 0.25 - 0.5 & & 3 \\
\text{Fine sand} & & 0.125 - 0.25 & & 5 \\
\text{Very fine sand} & & 0.064 - 0.125 & & 7 \\
\text{Coarse silt} & & 0.032 - 0.064 & & 9 \\
\text{Medium silt} & & 0.016 - 0.032 & & 10 \\
\text{Fine silt} & & 0.008 - 0.016 & & 11 \\
\text{Very fine silt} & & 0.004 - 0.008 & & 12 \\
\text{Coarse clay} & & 0.002 - 0.004 & & 13 \\
\text{Medium clay} & & 0.001 - 0.002 & & 14
\end{align*}
\]

TABLE 1 SEDIMENT GRADINGS

A significant feature of estuaries is the wide range of sediment sizes found in them. These sediments are sifted and sorted by the tidal currents.

In the main channels bed stresses are usually too high to allow the finer materials to accumulate although they may settle temporarily at slack water. Only coarse sand and gravel can exist as permanent deposits in these high energy regions. Along the shallow margins of the estuary, and further upstream, the tidal currents are too weak to move the sand and either no sand is transported there or it is covered by silt or clay to produce characteristic mud flats. These mud flats are colonised by various forms of marine life and become the feeding grounds of birds. If conditions are suitable the level of the mud flats rises and eventually a salt marsh develops.

The study of sediment transport generally is a very difficult problem. The particular study of sediment transport in estuaries is especially complicated because:

1. The water movements are continually changing with the rise and fall of the tide;
2. The wide range of sediments present in suspension and on the bed;
3. The absence of certain sediments in some parts of the estuary leading to unsaturated sediment loads in the water.

Although it is not possible to predict precisely how any single type of sediment will behave in the estuary, the recent advances in numerical modelling do enable some information to be obtained to give guidance to engineers and environmentalists for assessing the impact of engineering works.
In the following report various methods are described which have appeared in the engineering journals and proceedings of conferences over the last few years. The emphasis is on the formulation of models and applicability rather than on the details of particular numerical schemes used to obtain the solutions. We have tried to treat the methods in a systematic way starting with those appropriate for bed load transport and ending with models for studying fine silt and mud transport in suspension. There is inevitably some overlap between the modelling techniques if fine sand is concerned but it is hoped that the limits of applicability of any particular model will be clear. The author has attempted to fit all the relevant papers and reports known to him into this review, and apologises for any omissions which may have occurred through oversight.

2 POTENTIAL LOAD MODELS

The simplest type of sediment transport model is essentially a single equation representing conservation of bed material.

\[ \frac{\partial m}{\partial t} + \frac{\partial}{\partial x}(T_s) = 0 \]

where \( m \) (kg/m²) is the quantity of material on the bed and \( T_s \) (kg/sec/m width) is the sand transport. The basic assumption for this type of model is that the flow is saturated with sediment, which means that the flow is carrying the maximum sand transport that can be maintained for the given hydraulic and sedimentary conditions. Under saturated conditions the transport can be calculated from one of the many sediment transport laws to be found in the literature. An appraisal of available methods is given by White et al (1973). The flow parameters (water depth, mean velocity and shear velocity) required for the transport calculation could be obtained from measurements in a physical model but it is usually quicker and cheaper to generate this data on a regular grid from a separate numerical model of water movements.

The sediment carrying capacity of flow increases significantly for high water velocities -- typically in proportion to the fourth power. This means that the flow will tend to pick up material from the bed when it accelerates and to deposit excess material when it decelerates. If the flow is always saturated with sediment the differences in transporting capacity must define the quantity of material picked up or deposited on the bed. This is the basis for the potential load model. The computer is only used because it is many times faster than calculating by hand. In this way the sand transport calculation can be made for hundreds of points and repeated at intervals to define the variation of sand transport over the estuary and through the tidal cycle.

The potential load model is naturally most suited to situations where the bed material is narrowly graded and where there is an adequate supply of erodible material on the bed to maintain the saturated load. These conditions are more often met in rivers and it is in such situations that potential load models have been found most successful. See for example Cunge and Perdrew (1973), de Vries (1976), Thomas and Prasuhn (1977) and Bettens and White (1979). An example of the results from the last of these is given in Fig 1.

Lepetit and Hagues (1978) have extended the modelling approach used in river studies, to simulate 2-dimensional local scour around a jetty in a steady flow. The model is quasi-steady and uses a perturbation technique to feed the changes in depth back into the flow. Transport is calculated from a saturated bed load sediment law and bed changes calculated from the 2-dimensional form of equation 1, for conservation

### TABLE 3 STATE OF KNOWLEDGE ABOUT PHYSICAL RELATIONS

<table>
<thead>
<tr>
<th>Relation</th>
<th>Model</th>
<th>Method of studying</th>
<th>Quality of relation</th>
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<tbody>
<tr>
<td>Longitudinal dispersion</td>
<td>( D_0 )</td>
<td>SM</td>
<td>Theory/L/F</td>
</tr>
<tr>
<td>Lateral diffusion</td>
<td>( D_n )</td>
<td>SM</td>
<td>Lab/Field</td>
</tr>
<tr>
<td>Vertical diffusion</td>
<td>( D_z )</td>
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<td>Lab</td>
</tr>
<tr>
<td>Sand settling velocity</td>
<td>( w_s(D) )</td>
<td>S (sand)</td>
<td>Field</td>
</tr>
<tr>
<td>Advection deficit</td>
<td>( a )</td>
<td>S</td>
<td>Field</td>
</tr>
<tr>
<td>Profile factor</td>
<td>( s_0 )</td>
<td>S</td>
<td>Theory</td>
</tr>
<tr>
<td>Saturation S/T law</td>
<td>( T_s )</td>
<td>C_M</td>
<td>Field</td>
</tr>
<tr>
<td>Mud settling velocity</td>
<td>( W_0(c) )</td>
<td>M (mud)</td>
<td>Lab/Field</td>
</tr>
<tr>
<td>Deposition stress</td>
<td>( T_d )</td>
<td>M</td>
<td>Lab</td>
</tr>
<tr>
<td>Erosion stress</td>
<td>( T_d(q) )</td>
<td>M</td>
<td>Lab</td>
</tr>
<tr>
<td>Erosion rate</td>
<td>( M )</td>
<td>M</td>
<td>Lab?</td>
</tr>
<tr>
<td>Consolidation rate</td>
<td>( M_0 )</td>
<td>M</td>
<td>Lab?</td>
</tr>
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7 ACKNOWLEDGEMENTS

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The author is grateful for the helpful comments from many colleagues at HRS and in particular to Dr P. Better who carefully read through the final draft and made most useful suggestions.
5 STATE OF SEDIMENT TRANSPORT MODELLING

From this review of published information it is clear that the expertise already exists to formulate and develop the appropriate numerical techniques. Furthermore, computers are becoming more powerful and machines exist which can cope with the escalating number of calculations involved as the models become more sophisticated. The main factor limiting the advance of numerical sediment transport models is probably the uncertainties that exist about the physical relations which have to be fed into the models. Table 3 shows a list of the main relations that influence sediment transport in estuaries. The quality of the relations is graded from very good to poor merely to give a relative guide, but hopefully it will also inspire some researchers to improve the situation.

6 CONCLUSIONS

Many numerical models for studying sediment transport have been presented in recent years but very few are suitable for application to estuaries. If the author’s experience is typical, it is likely that many engineering studies have been made that have not been published because of pressures of other work or because the models were not fully calibrated or verified. From a scientific point of view an unverified model is of very little value. However, this is not necessarily true in an engineering sense provided the modeller or engineer recognises the shortcomings of the model and takes care in interpreting the model results. Results should not be taken on their face value but used to supplement the experience of the engineer, and to pinpoint the areas that the engineer should concentrate his attention on. Used in this way an unverified model is a valuable tool. For this reason there is a need for modellers and engineers to publish more readily their sediment transport case studies. Other workers in the field will appreciate the reason for limitations in such studies and the ensuing discussion can only serve to improve sediment models generally in the long-term.
Ouss proved most successful because the bed contained an adequate supply of erodible sand and inertial lag effects were small. Using a model of the river it was possible to reproduce the evolution of bed level profiles over a period of two years (Fig 3) and predict the changes which would occur following the construction of a tidal barrage and for extracting fresh water. A second application to a 2-dimensional area in the south east corner of the Wash was less satisfactory. The complicated nature of outfall channels prevented the collection of enough data to calibrate the model properly. Nevertheless it was still possible to use the model to predict the changes in sediment transport patterns which could occur following the construction of reservoir schemes.

in the Port is apparently caused by sediment brought down by the fast flowing Esmeraldas River in the wet season. Some of this settles into the underlying salt wedge in the lower estuary and eventually reaches the bed. A special 2-layer, 2-dimensional flow model was constructed to represent the river flow and the salt wedge, and a mud transport model was constructed for the surface layers. It was not possible to collect enough data to calibrate the model properly and as a consequence it was necessary to make sensitivity tests on the relative significances of turbulent mixing, settling, and entrainment parameters which controlled the amount of material dropping into the lower layer. The model was used to predict changes in suspended solids concentration and the settlement into the salt wedge and to the bed which would occur if a new harbour was built.

The next application by the Hydraulics Research Station was made to provide information about the lateral distribution of siltation in the Brisbane River to complement the longitudinal distribution of siltation predicted by the HRS multi-layer model. This work was commissioned by the Port of Brisbane Authority. An outline of the study was presented by Odd and Baxter (1980) and the particular details of the 2-dimensional model are described in Ref 14. The 2-dimensional model was run under steady peak flood and sheet flows on the principle that deposition under these conditions will identify areas where mud could accumulate. The model was verified against cores collected from areas identified by the model to be muddy or not muddy (Fig 18). The siltation rate was calibrated by reference to known dredging rates under existing conditions and the model used to predict areas and rates of siltation in the new docks.

The most recent application was to study siltation aspects of proposed engineering works in the Conway Estuary (Ref 15). The 2-dimensional models were in this case run on several tidal cycles and a sedimentation was governed by relation 31. There was no information available on the erosion properties of the Conway mud and consequently the effect of erosion was taken into account by choosing a critical stress for the accumulation of deposits. This critical stress was determined by comparing the maximum bed stress contours (predicted by the model) against the existing muddy areas. The same critical stress was subsequently used to identify the possible advance or retreat of the muddy areas following the construction of works. There were no siltation rates or dredging records available from the estuary to calibrate the model. Accordingly siltation rates in the developed stage were presented in the form shown in Fig 19 as ratios relative to the existing (but unknown) siltation rate at the same point. This information was used by environmentalists to assess the impact of the works on the ecology as well as by the engineers to assist in the design of works.
on the computer but there is also the extra difficulty in defining the properties of the bed over an area. To date the only known models for two-dimensional areas are in fact solutions of the depth integrated equation for the mud concentration, $\frac{\partial \phi}{\partial t} + \nabla \cdot (\vec{v} \phi) - \nabla \cdot (\Delta \phi \nabla) = \frac{\partial}{\partial z} \left( \frac{\partial \phi}{\partial z} \right) + S$.

(33)

where $S$ is defined by relations of the form given in equations 31 and 32. The main difference between this equation and equation 21 is that no profile parameters are considered necessary when very fine sediments are involved. Models based on equation 33 are therefore mainly valid in situations where the suspended material is well mixed in the vertical (concentrations $< 5000$ mg/l) and when lateral effects are more important.

The only published model of this type (Aristarchus and Krone (1976)) makes use of a finite element technique. The rate of exchange between the boundary and the bed was simulated by relations like those given in equations 31 and 32. The model was tested by comparing predicted siltation against siltation measured in a flume experiment (Fig. 17). The experimental set up consisted of a permeable barrier to create non-uniformities in a steady flow. Siltation occurred in the shelter of the barrier. The potential of the model was demonstrated by considering various alternatives for a hypothetical harbour problem.

3 SUSPENDED SAND TRANSPORT MODELS

The potential load model is not suitable for studying load transport in estuaries where there is not a continuous supply of erodible material on the bed. The reason is that that sort of model cannot take into account where the water carrying the sediment has been nor how much sediment is actually being carried by the flow. To do these requires a different sort of model based on conservation principles which simulate the sediment transport in terms of a suspended solids concentration. The erosion or deposition of material on the bed can then be assumed in the model depending on whether the actual load is less or greater than the saturated load which would obtain under steady, uniform flow conditions at the same values as the instantaneous flow. This assumption effectively ignores the differences in turbulence which occur in accelerating or decelerating flow. Under these circumstances the suspended solids concentration, $c$, (kg/m$^3$) satisfies,

\[ \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z} = \frac{\partial}{\partial x} \left( \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial c}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial c}{\partial z} \right) + S \]

(2)

where $u, v, w$ are the velocity components $x, y, z$ are space co-ordinates, with $z$ vertically upwards $D_x, D_y, D_z$ are the (turbulent) diffusion coefficients $w_s$ is the settling velocity $S$ is the sink or source term representing erosion or deposition of material on the bed $t$ is time.

Most solutions to be found in the literature are for special cases of this equation. The earlier solutions by Schmidt (1925) and Lane et al (1941), and later Hunt (1965) are essentially geared to providing insight into the vertical structure of the suspended solids profile. These assume one-dimensional, uniform, steady flow conditions for which equation 2 reduces to

\[ w \frac{\partial c}{\partial z} + \frac{\partial (D \frac{\partial c}{\partial z})}{\partial z} = 0 \]

(3)
This equation represents the equilibrium profile obtained as a balance between settling and vertical diffusion due to the turbulence. It is important to appreciate that equilibrium defined in this way does not mean saturation. Indeed, the sediment load can be in equilibrium if the bed is not mobile, even when the flow is under saturated with sediment. Integration with respect to \( z \) and the application of a boundary condition of zero flux of sediment at the free surface (and implicitly also at the bed) yields the governing equation

\[
wc + D_{zz} \frac{dwc}{dz} = 0
\]

(4)

This can be integrated further if the vertical structure of the diffusivity is prescribed. These profiles have proved valuable in the understanding of sediment transport but they are not relevant to the unsteady, unsaturated flow conditions which are the main concern here, so these special solutions are not considered further. Graf (1971) is a good source of additional information on these solutions.

A class of solutions which have more relevance to estuaries have been presented by Kalinske (1940), Dobbins (1943), Mei (1969) and Lean (1980), for unsteady and uniform or steady and non-uniform conditions governed respectively by the equations

\[
\frac{\partial c}{\partial t} - \frac{\partial}{\partial x} (D \frac{\partial c}{\partial x}) + \nu \frac{\partial^2 c}{\partial z^2} = 0
\]

(5)

\[
\frac{\partial wc}{\partial t} - \frac{\partial}{\partial x} (D \frac{\partial wc}{\partial x}) + \nu \frac{\partial^2 wc}{\partial z^2} = 0
\]

(6)

These equations are mathematically the same when \( \nu \) is constant. The first represents a concentration changing with time following a change in the magnitude of a uniform flow, while the second represents the concentration change as a function of position as might occur for example when clear water flows from an area with an incredible bed into an area where erosion can commence. Solutions of these equations provide information about the time or distance of travel required for the sediment concentration to adapt to changes in the flow conditions.

The assumption of constant eddy diffusivity permits analytic solution of these equations. The diffusivity normally used is

\[
D_z = k \frac{u_v}{d}
\]

(7)

which is the depth averaged value of the parabolic eddy viscosity

\[
\nu_e = k (1 - z/d)
\]

(8)

consistent with the logarithmic velocity profile. \( k \) is the Von Karman constant. Asmussen and Runger (1970) present experimental evidence that supports this assumption. The exact solutions, which involve the use of Laplace Transform, or similar, may be expressed in the form of infinite series. Mei (1969) recognised that an approximate solution, valid for small times or distances of travel, could be obtained from the expansion of the Laplace Transform for large values of the transform parameter. Under most conditions this expansion is valid for distances of the order of twenty water depths or the equivalent in a time dependent situation. Lean (1980) proposed an alternative bed boundary condition and the present author has reworked Mei's solution for this case. This solution is used in the sediment transport model proposed in the next section.

The solution of the unsteady equation 5 requires an initial condition at \( t = 0 \) and boundary condition at \( z = 0 \) (the bed) and \( z = d \) (the free surface). The surface boundary condition is clearly zero vertical flux of sediment viz

4.2 2-dimensional mud transport models

Boonville (1976) has applied a very similar 2-layer model with a constant lower layer to the Georne.

Experience in the use of these 2-layer models at the Hydraulics Research Station identified the need for and led to the development of numerical techniques for describing the vertical structure of the flow with many layers. A model of this type that has been applied to study the situation in the estuaries of the Port of Bribane. The essentials of the model are described by Rodger (1976). The model uses relations of the form in equations 31 and 32 for exchanges of mud between the bed and the lower layer, and includes a layered description of the bed to keep account of the age of deposits. Exchange of material between the layers is simulated as a balance between settling, upwelling and turbulent mixing as for the earlier 2-layer model but in addition the rate of turbulent mixing is adjusted dynamically to allow for stratifications in the flow as described by Odd and Rodger (1979).

Watanabe et al (1978) propose a different method of representing the vertical flux using a finite element approach. At present this model is limited to uniform flow but the paper contains some flume data which could serve as a useful check on model response to changes in flow.

A model to study a special fluid mud problem in the River Avon was developed at the Hydraulics Research Station (HRS Ex 702 (1975)). Mud concentrations in this river exhibit the same type of behaviour described by Parker and Kirkby (1979) for the Severn Estuary into which the river in fact flows. The model was designed to simulate three distinct phases of suspension: a) a static bed layer with a uniform concentration of 150 000 ppm, a fluid mud layer with concentrations varying between 75 000 and 10 000 ppm and a suspended mud layer with a variable concentration distributed uniformly through the layer. The flow and transport were calculated dynamically using a finite difference technique. The exchange relations between the layers were similar to those used by Odd and Owen in their 2-layer model. The erosion rate was chosen during calibration to produce the correct thickness of static layer observed in the river. The model was used to study the influence of a proposed barrage on the mud regime of the River Avon.

The extension of the numerical techniques used in the 1-dimensional models to study mud transport in 2-dimensional situations still has not yet reached the stage for presentation in the literature. One reason is undoubtedly because of the high cost of 2-dimensional layered models.
It is well known (Azizbahlou and Kroone (1976) and Thorn and Parsons (1980)) that erosion does not commence until the bed shear exceeds a certain critical value, \( \tau_b \), and that the erosion rate seems to be proportional to the excess shear. However, this critical stress varies with the density of the exposed mud surface, which in turn varies with time. As mud is eroded, a more consolidated (and hence denser) mud is exposed which requires a corresponding higher bed stress to put it into suspension. Parthenaides (1965) proposed representing erosion as

\[
S_e = M \left( \frac{\tau_b}{\tau_c} - 1 \right) \quad \tau_b = \tau_c
\]  

(32)

in which the erosion rate \( M \) has to be prescribed from experiments in addition to the critical erosion stress.

The fundamental equation governing the transport of mud in suspension is identical to equation 2 with the source/sink term given by equations 31 or 32. The special nature of these boundary conditions at the bed precludes the formulation of analytic solutions thus all the models described below are numerical. The models reported in the literature are often designed to meet the particular requirements of certain problems. In the following the models are separated into those which involve just one horizontal dimension in which lateral effects are neglected, and those which take into account lateral as well as longitudinal variations. Either sort of model can include a description of vertical effects.

### 4.1 One-dimensional mud models

One-dimensional models are designed for estuaries or tidal channels where lateral variations are not important. Unfortunately, these situations are likely to be more affected by salinity effects insofar that secondary currents can be created in the vertical plane. Even if the mud appears to be well mixed through the water column, the presence of non-logarithmic velocity profiles necessitates the use of 2 or more layers in the vertical to simulate the mud transport.

The earliest model of this type presented by Odd and Owen (1972) used 2 layers to simulate the mud transport processes in the Thames Estuary. The lower layer was given a fixed sediment thickness and the flow and concentration calculated in each layer using a finite difference method. The exchange of material between the lower layer and the bed was based on relations of the type given in equations 21 and 22 in which the critical stresses were estimated from laboratory experiments.

Exchange of material between the upper and lower layer was computed from the net flux of mud due to settling, upwelling, and vertical mixing by turbulence. The model was verified by comparing predicted and observed concentrations. The results showed very good agreement between areas of deposition predicted by the model and the existing muddy reaches of the river (Fig 16). The application of the model to study the effects of a tidal barrage clearly demonstrated its value for the engineer.

This model was subsequently improved to allow the interface between the layers to adjust dynamically to the flow and salinity variations. It has been applied to study tidal flows, saline intrusion and sediment transport in the Rotterdam Waterways. A series of sensitivity tests was carried out to demonstrate the effect of altering mixing and entrainment coefficients on the pattern of saline intrusion. Comparison with two independent sets of field observations showed that the model simulated all the gross features of the pattern of saline intrusion in the waterways for average and low flow values with the same set of model parameters. The model also simulated correctly all the important periodic variations in the concentrations of suspended mud in both layers during a period of low flow values.

\[
w + D \frac{\partial S}{\partial z} = 0 \quad z = d
\]  

(9)

There are two possible conditions at the bed which admit analytical solution. Firstly, one could assume that the concentration \( c_0(t) \) the bed responds instantaneously to the changing flow conditions.

That is

\[
\dot{c}_0(t) = c_0(t) = b_0 \dot{c}_0(t)
\]  

(10)

where

\[
c_0(t)
\]

is the concentration of the equilibrium profile at the bed when the flow is saturated with sediment

\[
\tilde{c}_0(t)
\]

is the depth averaged value of this profile and

\[
b_0 = \frac{c_0(t)}{\tilde{c}_0(t)}
\]

is a profile factor

(11)

This is a much more realistic condition than that implicit in a potential load model which assumes that the full load responds instantaneously. However, the condition still implies an infinite rate of exchange of material at the bed at \( t = 0 \) (or at \( x = 0 \) in the non uniform version).

Lee (1980) assumed that the rate at which material is entrained into the flow is the quantity which responds most readily to changes in flow. In this case the boundary condition at the bed would be

\[
D_s \frac{\partial S}{\partial z} \bigg|_{z=0} = (\dot{D}_s \Delta c_s)_s \bigg|_{z=0}
\]  

(12)

or, from equation 4, the net vertical flux at the bed is prescribed as

\[
w \frac{\partial \tilde{c}_0}{\partial z} \bigg|_{z=0} = w \frac{\partial \tilde{c}_0}{\partial z} - \tilde{c}_0
\]  

(13)

This provides an alternative boundary condition to condition 8. The asymptotic form of solutions to equation 5 for initial concentration \( \tilde{c}_0 \) are

\[
\dot{c}_0(\tau) = \frac{2}{\tau} \frac{\partial \tilde{c}_0}{\partial \tau} \left( \tau - \tilde{c}_0 \right) \text{erf}(\tau^2 + \tau) + \frac{e^{\tau^2 \text{erf}(\tau^2 + \tau)} - \frac{e^{-\tau^2 \text{erf}(\tau^2 - \tau)}}{\sqrt{\pi}}}{\tau^2}
\]  

(14)

and

\[
\dot{c}_0(\tau) = \frac{2}{\tau} \frac{\partial \tilde{c}_0}{\partial \tau} \left( \tau - \tilde{c}_0 \right) \text{erf}(\tau^2 + \tau) + \frac{e^{\tau^2 \text{erf}(\tau^2 + \tau)} - \frac{e^{-\tau^2 \text{erf}(\tau^2 - \tau)}}{\sqrt{\pi}}}{\tau^2}\text{erf}(\tau^2 + \tau)
\]  

(15)

where

\[
R = \frac{w \Delta D_s}{\Delta h}
\]

(16)

\[
r = \frac{\Delta D_s}{\Delta h}
\]

(17)

\[
o = \frac{D_s}{\Delta h}
\]

(18)

\[
\frac{\beta}{d} = \frac{\beta}{d}
\]

(19)

for the bed concentration boundary condition (Mei) and the bed entrainment boundary condition (Lee) respectively. These solutions are valid when \( r < 1 \), i.e. for times while water flows over distances equal to 10 to 100 water depths.

There is another class of special analytic solutions which are related to the plume models used for thermal or sewage pollution studies. These models are usually depth integrated and are designed for situations
Fig. 4. Comparison of observed plume structure in Apalachicola Bay. (From Fig. 6 in Wilson (1979) courtesy Academic Press Inc. (London) Ltd)

when the lateral spread of the plume is considered to be more important than the vertical profile. Wilson (1979) describes a model of this sort with an integral solution representing the balance between advection with the ambient current, lateral and longitudinal diffusion and settlement of particles of various sizes. The model is presented as a tool for studying sediment plumes produced from dredging losses. A typical comparison between results from the model and observations is reproduced in Fig. 4. Another integral solution for studying plumes from continuous discharges is presented by Christodoulou et al. (1974), and Chen et al. (1978) present a special model which computes bed changes in a reservoir due to settlement of suspended material carried in by the incoming water. The water flow in the latter case is obtained from the classical structure of a turbulent jet, and the suspended solids concentration is computed using a finite difference scheme. Chen et al. (1978) also present some experimental results which could be of value in testing models of this type.

Fig. 5. Comparison of observed and calculated concentration profiles. (From Fig. 3 in Valencia and Finlayson (1973) courtesy International Association for Hydraulic Research)

Fig. 15. Development of suspension structures over one tidal cycle. (From Fig. 3 of Parker and Kirby (1977), courtesy BHRA Fluid Engineering)

near the bed to balance the seaward drift entrained by the surface water. Special techniques will again be required to study the transport of mud in these situations.

The simulation of mud transport requires a more comprehensive knowledge of the physical properties of the sediment than was required for studying sand transport. The properties of mud, such as settling velocity, erosion stress, deposition stress, and erosion rate, have been the subjects of many experimental studies over the years (Parnetaides (1965), Krone (1962), Owen (1971), and Thorn and Parsons (1980)).

It appears that the settling velocity of cohesive material in suspension for concentrations less than 3000 m/g/l is proportional to the concentration with a proportionality factor of between 0.001 and 0.002 m/s per mg/l. This is because there is a greater likelihood of collisions when there are more particles present. Above 5000 m/g/l the settling velocity does not increase linearly because of hindered settling.

If the local bed stress is too high mud particles or flakes which drop onto the bed cannot settle. Krone (1962) postulated that the probability, $P$, that flakes stick to the bed increases linearly from zero as the bed stress, $r_b$, falls below the critical deposition stress, $r_d$, thus

$$P = (1 - \frac{r_b}{r_d})$$  \hspace{1cm} \text{if } r_b < r_d \tag{30}$$

The net deposition rate can then be expressed as

$$S_y = P_n \bar{v} = \bar{v}(1 - \frac{r_b}{r_d})$$ \hspace{1cm} \text{with the value of } r_d \text{ determined from tests carried out on mud from the estuary under investigation.} \tag{31}$$
Areas with $c_s < 1$ are potential areas of erosion but erosion only occurs if there is mobile material on the bed. Note how the finer sand takes longer to adapt to the new flow conditions.

Figure 14 shows the bed changes predicted in the model for two representative sediment sizes, together with the corresponding changes from a potential load model. The idealised channel was designed to emphasise the limitations of potential load models in estuarine conditions, namely:

1. Erosion may not in fact occur in an area of potential erosion if there is no erodible material on the bed — see (1) in Fig 14.
2. Deposition may not in fact occur in a region of potential deposition if the sediment load of the approaching water is insufficient to saturate even the slower flow — see (3) in Fig 14.
3. Erosion can even occur in an area of potential deposition if the sediment load of the approaching flow is very low — see (3) in Fig 14.

The results also show that:

4. Erosion in the new model can occur in an area of uniform flow — see (4) in Fig 14.
5. The new model distributes deposits downstream and the finer sediment is deposited over a greater distance due to its lower settling velocity — see (5) in Fig 14.

![Fig 14. Calculated bed changes in test channel](image)

The fact that the new model can simulate these properties of sand transport makes it a much more powerful tool than potential load models. The next step will be to calibrate and verify the model in a real situation. It is hoped that this will be possible in the near future using the Conwy Estuary to enable a direct comparison to be made against the results of the potential load model previously used on that estuary. In addition there are plans to extend the model to deal simultaneously with several sand fractions.

### 4 MUDB TRANSPORT MODELS

The relatively high density of sand grains produces a vertical structure in the sediment load which has to be taken into account in the sediment modelling. The most pronounced structure arises when the largest sand fractions are involved. At the other extreme one might expect the very fine silt and clay materials to form even distributions through the

### 3.1 Numerical sand transport models

Although the special solutions described above provide insight into the sediment transport processes they lack many of the factors which are important in estuaries, namely:

1. the combination of non uniformity with unsteadiness
2. variable supply of erodible material
3. lateral as well as longitudinal variation.

The inclusion of these factors completely precludes any possibility of analytic solution and leads to the need for numerical models.

The simplest form of numerical sediment transport model takes the form of finite difference or finite element solutions of the approximations to equations 5 and 6. ApSimon and Rutter (1970) and Yalin and Finlayson (1973) present models of this type. The advantage of seeking numerical solutions is that more realistic eddy diffusivities and velocity profiles can be incorporated. Yalin employs an eddy diffusivity equal to the parabolic eddy viscosity (eq 8) consistent with the logarithmic flow profile. The model was tested against experimental measurements (Fig 5).

Although the model is not immediately relevant to estuaries there is no inherent difficulty in extending the numerical techniques to non uniform, unsteady conditions.

Kerssen et al (1979) have developed a multi-layer, 1-D model of this type which allows non uniform cross-section to be considered but it has apparently only been applied under steady flow conditions. The basic flow equation

$$\frac{dU}{dx} + \frac{dD}{dx} = -\frac{U^3}{C R_h}$$

where

- $U$ is water depth
- $D$ is elevation of the bed
- $C$ is Chezy coefficient
- $R_h$ is the hydraulic radius

is used to calculate the magnitude of the depth averaged current $U(x)$. A logarithmic profile is assumed for the vertical structure of the flow. The suspended sediments concentration is computed from the 1-dimensional form of the sediment concentration equation 2 using non uniform vertical grid to give greater accuracy near the bed. Kerssen et al (1979) assumed the turbulent diffusivity, $D_t$ to equal the parabolic eddy viscosity (eq 8) appropriate for logarithmic flow. The boundary condition at the bed is taken to be the equilibrium concentration, equation 10, which would occur at the instantaneous flow conditions. The solution simulates the transient evolution of the equilibrium profile from a non equilibrium condition. The model was tested against infill rates measured in a gas pipeline trench in the Western Scheldt. Some results from the paper are reproduced in Fig 6.

Koutrous and O'Connor (1980) present a similar model, but use the alternative bed condition (12) instead of prescribing an equilibrium concentration at the bed. Their paper is mainly concerned with the mathematics of finite element versus finite difference solutions. They show that the qualitative structure of the results are not sensitive to the computational method used but the full finite element model proved to be more expensive than the preferred hybrid finite element/
finite difference model. The validity of the model was verified for classical diffusion solutions.

Farmer and Waldrip (1977) have presented a similar model which has been applied to study a true three-dimensional situation. This model uses finite difference techniques to solve the steady state version of equation 2 in approaching the full estuary conditions. In the model trajectories of particles from a river flow are computed under the influence of water currents, settlement and vertical diffusion to simulate the delta formation in the sea. The model was designed to study the Mississippi Delta. The model incorporates a bottom boundary condition that includes a probability relation that a particle is reflected back into the flow and there is a scouring rate coefficient.

Another model for studying three dimensional problems has been reported by Sengupta et al. (1978). This is formulated as a full 3-dimensional model including time variations. The proposed boundary condition at the bed contains both concentration and diffusion flux at the bed and two parameters for the bed depositional rate and for the stickiness of the sediment. The model has been applied to Biscayne Bay, Florida. However Sengupta, et al recognised the lack of an adequate data base for verifying the model and consequently it was tested by studying the influence of the settling velocity and bed deposition parameter on the concentration profiles arising from an initial line source of suspended sediment. The results, reproduced in Fig. 7, show that the surface concentration is more sensitive to settling velocity than to the bed deposition parameter. The reason for this is that although changes to the settling velocity and the bed deposition parameter are expected to have the same effect at the bed, the settling velocity also affects the feed of sediment through the water column.

Although none of the models described so far have been applied to real estuary conditions there is no technical reason why this should not be done. The main problems preventing this at present seem to be the high expense of running 3-dimensional models and deficiencies in our knowledge of sediment transport processes in estuaries. In an attempt to gain an understanding of the consequences of unstratified flow in estuaries the present author has proposed a 2-dimensional, depth integrated model. This type of model requires special provision to take into account the vertical profile effects of the sediment concentration.
The depth averaged concentration $C(x,y,t)$ satisfies the depth integrated form of the equation 2 which may be written as:

$$\frac{\partial}{\partial t}(D_n \frac{\partial C}{\partial y}) + \frac{\partial}{\partial x} (D_n \frac{\partial C}{\partial x}) + \frac{\partial}{\partial n} (D_n \frac{\partial C}{\partial n}) = \rho_w (C_0 - C)$$

where

- $D_n$ is a longitudinal dispersion coefficient due to the vertical profile.
- $D_h$ is the lateral (turbulent) diffusion coefficient.
- $(x,n)$ are natural co-ordinates in the direction and normal to the flow.

Nihoul and Adam (1975) proposed a model similar to this. The parameters $\alpha$ and $\beta_n$ are introduced to account for the vertical concentration and velocity profiles. For example:

$$\alpha = \frac{1}{q} \int_0^1 \frac{\partial C}{\partial y} dy$$

where

$$q = (v^2 + \nabla U)^{1/2}$$

is the horizontal water speed, represents the factor required to recover the true transport of sediment from the product of depth averaged quantities. Since high concentrations occur near the bed it follows that $\alpha < 1$, and $\beta_n > 1$.

Miles et al (1980) have analysed sediment transport measurements from the Conway estuary in terms of $\alpha$ and $\beta_n$. The results for $\alpha$ in Table 2 are the most consistent on both a station basis and overall for the estuary and the mean is in good agreement with theoretical values from Sumer (1977) for conditions typical of flow in the Conway estuary. The scatter in $\beta_n$, illustrated by the standard deviation, is greatest for stations 1 and 10 where no observations were made lower than 0.5m above the bed, and the omission of stations where less information was collected gave a mean $\beta_n$ value of 0.6 with a standard deviation of only 2.4. This value is in good agreement with theoretical values.
\[ \delta_3 = 2R(1 - \exp(-2R))^2 \]  

(21)

from the exponential equilibrium profile of Lane et al. (1941) for conditions typical of the flow in the Conwy Estuary.

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<th>Station</th>
<th>Date</th>
<th>Number of useful profiles</th>
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<th>Standard deviation</th>
<th>Mean ( \delta_3 )</th>
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<tr>
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* Profile calculated from measurements at minimum depths 0.5 m above the bed

**TABLE 2** PROFILE FACTORS FOR THE CONNY ESTUARY

The results show that it is possible to obtain sensible profile factors from field measurements and, although the relations found are valid only for the Conwy, the methods involved could be applied to any site. With the empirical method of prescribing \( \alpha \) and \( \delta_3 \), this model has the basic features of studies on sediment transport in estuaries. It has advection by currents, dispersion in the direction of flow due to the vertical profile and lateral by turbulence. Deposition or erosion takes place depending as to whether the instantaneous sediment load exceeds or falls short of the saturated load and, if required, erosion may be prevented if there is no sediment of the appropriate size available on the bed. A shortage of material on the bed would be reflected in a low concentration of suspended solids being advected away by the flow.

The most unsatisfactory aspect of this model is the use of the settling velocity as the main scaling factor for the exchange rate of material between the flow and the bed. A better approximation for this exchange rate can be determined from the analytical solutions (14) or (15) for transient conditions. The rates of exchange of sediment at the bed

\[ F_{se}(r) = -\left(\frac{D_1 w_1 C_{1b}}{z} + \frac{w_2 C_{2b}}{z}\right) \]  

are

\[ F_{se}(r) = \frac{1}{2} \left( \frac{w_1 C_{1b}}{z} \right) \left( \frac{1}{w_1} \right) \]  

and

\[ F_{se}(r) = \frac{w_2 C_{2b}}{z} \]  

for the boundary conditions of Mei and Lean respectively. The nature of these relations is shown in Fig 8 for a typical flow condition. Since relation 25 implies an infinite flux of sediment at \( t = 0 \) the second formulation is favoured in the following. If preferred, the analysis could be repeated for the other case.

Fig 8 shows that the rate of exchange of sediment at the bed varies considerably over times of the order of timesteps normally used in numerical models. Accordingly it is advisable to integrate equation 26 over a model timestep. That is the vertical flux of sediment, \( S_{se}(r) \), during the interval \( (0, t) \) is

\[ S_{se}(r) = w_1 C_{1b} - \frac{\partial C_{1b}}{\partial t} \]  

(27)

where

\[ \frac{\partial C_{1b}}{\partial t} = \frac{\partial}{\partial t}\left( \frac{w_1 C_{1b}}{z} \right) \]  

(28)

is the bed exchange scaling factor that incorporates the effects of the vertical structure and the lag time for the concentration profile to adjust to the changing flow conditions. The nature of \( \delta_3(r) \) is shown in Fig 9 for a typical sediment fraction and timestep.

A new sand transport model has been developed by the author at the Hydraulics Research Station using \( S_{se}(r) \) given in equation 27 to replace the source/sink term on the right hand side of equation 21. This is equivalent to assuming that the sediment load is in equilibrium with the flow, which is reasonable provided the flow is slowly varying in space and time. However it follows that the new method cannot be absolutely precise because the theoretical solutions 14 and 15 have a gradual transition while the approximate computation is calculated from the assumed equilibrium profile at the start of each timestep. However, results under uniform flow conditions shown in Fig 10 indicate that the errors involved are less than the differences which result from applying the two alternative boundary conditions 10 and 12. The advantages of the new model become apparent from the following results for the special case of a steady flow Q per m width along
\[ \delta_3 = 2R(1 - \text{exp}(-2R))^{-1} \]  
\[ \text{(21)} \]

from the exponential equilibrium profile of Lane et al. (1947) for conditions typical of the flow in the Conwy Estuary.

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</tbody>
</table>

* Profile calculated from measurements at minimum depths 0.5m above the bed

TABLE 2: PROFILE FACTORS FOR THE CONWY ESTUARY

The results show that it is possible to obtain sensible profile factors from field measurements and, although the relations found are valid only for the Conwy, the techniques involved could be applied to any site. With this empirical method of prescribing \( \delta_3 \) and \( \delta_0 \), this model has the basic features for studying sediment transport in estuaries.

It has advection by currents, dispersion in the direction of flow due to the vertical profile and lateral diffusivity by turbulence. Deposition or erosion takes place depending as to whether the instantaneous sediment load exceeds or falls short of the saturated load, and, if required, erosion may be prevented if there is no sediment of the appropriate size available on the bed. A shortage of material on the bed would be reflected in a low concentration of suspended solids being advected away by the flow.

The most unsatisfactory aspect of this model is the use of the settling velocity as the main scaling factor for the exchange rate of material in the flow and the bed. A better approximation for this exchange rate can be determined from the analytical solutions (14) or (15) for transient conditions. The rates of exchange of sediment at the bed

\[ F_2(r) = -(D_2 \frac{\delta_3}{\delta_0} + \frac{\delta_0}{\delta_3}) \]  
\[ \text{(24)} \]

are

\[ F_2(r) = \frac{1}{2} g_0 \beta (C_0 - \bar{C}_0)(\frac{1}{\sqrt{4\pi \tau}}e^{-\frac{r^2}{4\tau}} - \text{erfc}(r)) \]  
\[ \text{(25)} \]

and

\[ F_2(r) = \mu g_0 \beta (C_0)(1 + 2r^2) \text{erfc}(r) - \frac{2r}{\sqrt{\pi}} e^{-r^2} \]  
\[ \text{(26)} \]

for the boundary conditions of Mei and Lean respectively. The nature of these relations is shown in Fig 8 for a typical flow condition. Since relation 25 implies an infinite flux of sediment at \( t = 0 \) the second

Fig 8 shows that the rate of exchange of sediment at the bed varies considerably over times of the order of timesteps normally used in numerical models. Accordingly it is advisable to integrate equation 26 over a model timestep. That is the vertical flux of sediment, \( S_0(t) \), during the interval \( (0, t) \) is

\[ S_0(t) = \int_0^t F_2(r) g_0 \beta (C_0 - \bar{C}_0) dr \]  
\[ \text{(27)} \]

where

\[ g(r) = \frac{1}{\sqrt{4\pi \tau}}e^{-\frac{r^2}{4\tau}} \]  
\[ \text{(28)} \]

is the bed exchange scaling factor that incorporates the effects of the vertical structure and the lag time for the concentration profile to adjust to the changing flow conditions. The nature of \( g(r) \) is shown in Fig 9 for a typical sediment fraction and timestep.

A new sand transport model has been developed by the author at the Hydraulics Research Station using the bed exchange term given in equation 27 to replace the source/sink term on the right hand side of equation 21. This is equivalent to assuming that the sediment load is in equilibrium with the flow, which is reasonable provided the flow is slowly varying in space and time. However it follows that the new model cannot be absolutely precise because the theoretical solutions 14 and 15 have a gradual transition while the approximate computation is calculated from the assumed equilibrium profile at the start of each timestep. However, results under uniform flow conditions shown in Fig 10 indicate that the errors involved are less than the differences which result from applying the two alternative boundary conditions 10 and 12.

The advantages of the new model become apparent from the following results for the special case of a steady flow Q per m width along
Fig 9. Bed change factor

Fig 10. Theoretical and calculated sediment rates

Fig 7. Sediment particle concentrations versus lateral distance at the surface
(From Fig 4-1 in Sengupta et al. (1979); courtesy University of Miami)

The depth averaged concentration \( c(x, y, t) \) satisfies the depth integrated form of equation 2 which may be written:

\[
\frac{\partial}{\partial t} (cD) + \phi \left( \frac{\partial (cD \mu)}{\partial x} + \frac{\partial (cD \nu)}{\partial y} \right) = \frac{\partial}{\partial y} \left( \frac{cD \kappa}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{cD \kappa}{\partial x} \right) + \beta_w \left( C_n - 2 \right)
\]

where

- \( D_\parallel \) is a longitudinal dispersion coefficient due to the vertical profile
- \( D_n \) is the lateral (turbulent) diffusion coefficient
- \( x, y, t \) are natural co-ordinates in the direction and normal to the flow

Nihoul and Adam (1975) proposed a model similar to this. The parameters \( \alpha \) and \( \beta_w \) are introduced to account for the vertical concentration and velocity profiles. For example:

\[
\alpha = \frac{1}{Q(x, y, t)} \frac{d}{dx} \quad (23)
\]

where

\( Q(x, y, t) = (Q_1^2 + Q_2^2)^{1/2} \) is the horizontal water speed

represents the factor required to recover the true transport of sediment from the product of depth averaged quantities. Since high concentrations occur near the bed it follows that \( \alpha < 1 \), and \( \beta_w > 1 \).

Miler et al. (1980) have analysed sediment transport measurements from the Conway estuary in terms of \( \alpha \) and \( \beta_w \). The results for \( \alpha \) in Table 2 are the most consistent on both a station basis and overall for the estuary and the mean is in good agreement with theoretical values from Sumer (1977) for conditions typical of flow in the Conwy estuary. The scatter in \( \beta_w \), illustrated by the standard deviation, is greatest for stations 1 and 10 where no observations were made lower than 0.5m above the bed, and the omission of stations where less information was collected gave a mean \( \beta_w \) value of 2.0 with a standard deviation of only 2.4. This value is in good agreement with theoretical values
finite difference model. The validity of the model was verified for classical diffusion solutions.

Farmer and Walsh (1977) have presented a similar model which has been applied to study a true three-dimensional situation. This model uses finite difference techniques to solve the steady state version of equation 21 approaching the full estuary conditions. In the model trajectories of particles from a river flow are influenced by the influence of water currents, sediment and vertical diffusion to simulate the delta formation in the sea. The model was designed to study the Mississippi Delta. The model incorporates a bottom boundary condition that includes a probability relation that a particle is reflected back into the flow and there is a scouring rate coefficient.

Another model for studying three dimensional problems has been reported by Sen Gupta et al (1978). This is formulated as a full two-dimensional model including time variations. The proposed boundary condition at the bed consists both concentration and diffusion flux at the bed and two parameters for the bed deposition rate and for the stickiness of the sediment. The model has been applied in Bayou Bayou, Florida. However Sen Gupta, et al recognised the lack of an adequate data base for verifying the model and consequently it was tested by studying the influence of the settling velocity and bed deposition parameter on the concentration profiles arising from an initial line source of suspended sediment. The results, reproduced in Fig 7, show that the surface concentration is more sensitive to settling velocity than to the bed deposition parameter. The reason for this is that although changes to the settling velocity and the bed deposition parameter are expected to have the same effect at the bed, the settling velocity also effects the feed of sediment through the water column.

Although none of the models described so far have been applied to real estuary conditions there is no technical reason why this should not be done. The main problems preventing this at present seem to be the high expense of running 3-dimensional models and deficiencies in our knowledge of sediment transport processes in estuaries. In an attempt to gain an understanding of the consequences of unstratified flow in estuaries the present author has proposed a 2-dimensional, depth integrated model. This type of model requires special provision to take into account the vertical profile effects of the sediment concentration.

the idealised channel shown in Fig 11 which has depth variations d(x).

The bed of the channel is assumed to contain only three patches of mobile sediment. Clear water is fed into the left hand end of the model and the concentration along the channel was calculated using a source/sink term given by equations 27 and 28.

Figure 12 shows the calculated concentration, \( c_2 \), compared against the saturation concentration, \( c_s \), obtained from the simple transport relation

\[
T_c = u_s c_s = E u_s (K_e - U_{csp}) \text{ kg/m}^3/\text{m width}
\]

where \( u_{crit} \) is the threshold velocity for initiation of motion and \( E \) is a coefficient depending on the sediment size. The calculated concentrations in Fig 12 tend towards the saturated concentration from above (A) or below (B) depending whether the flow is up or over saturated. If the flow is over saturated, sand is deposited as the actual concentration drops. However, in the other case erosion is only permitted if there is mobile material on the bed. If the bed is hard the concentration in the model remains constant even if it is unsaturated (C).

Figure 13 shows an alternative method of viewing the problem using the saturation ratio \( c_s = c/c_s \). Areas with \( c_s > 1 \) will suffer deposition.

Fig 6. Measured and computed bed levels

From Fig 11 in Kerssen et al (1979); courtesy American Society of Civil Engineers.

Fig 7. Calculated concentrations along test channel

Fig 8. Calculated saturation ratio along test channel
Areas with $c_e < 1$ are potential areas of erosion but erosion only occurs if there is mobile material on the bed. Note how the finer sand takes longer to adapt to the new flow conditions.

Figure 14 shows the bed changes predicted in the model for two representative sediment sizes, together with the corresponding changes from a potential load model. The idealised channel was designed to emphasise the limitations of potential load models in estuarine conditions, namely:

1. Erosion may not in fact occur in an area of potential erosion if there is no erodible material on the bed — see (1) in Fig 14.
2. Deposition may not in fact occur in a region of potential deposition if the sediment load of the approaching water is insufficient to saturate even the slower flow — see (3) in Fig 14.
3. Erosion can even occur in an area of potential deposition if the sediment load of the approaching flow is very low — see (3) in Fig 14.

The results also show that:

4. Erosion in the new model can occur in an area of uniform flow — see (4) in Fig 14.
5. The new model distributes deposits downstream and the finer sediment is deposited over a greater distance due to its lower settling velocity — see (5) in Fig 14.

The new model is the result of a numerical sand transport model that allows non uniform cross-sections to be considered, but unfortunately has been applied under steady flow conditions.

4 MUD TRANSPORT MODELS

The relatively high density of sand grains produces a vertical structure in the sediment load which has to be taken into account in the sediment modelling. The most pronounced structure arises when the largest sand fractions are involved. At the other extreme one might expect the very fine silt and clay materials to form even distributions through the

Although the special solutions described above provide insight into the sediment transport processes they lack many of the factors which are important in estuaries, namely:

1. the combination of non uniformity with unsteadiness
2. variable supply of erodible material
3. lateral as well as longitudinal variation.

The inclusion of these factors completely precludes any possibility of analytic solution and leads to the need for numerical models.

The simplest form of numerical sediment transport model takes the form of finite difference or finite element solutions of the approximate equations 5 and 6. Aparna and Runn (1970) and Talin and Fialoycan (1973) present models of this type. The advantage of seeking numerical solutions is that modern computer facilities can be adequately employed for the sake of selected solutions. Talin employs an eddy diffusivity equal to the parabolic eddy viscosity (eq 8) consistent with the logarithmic flow profile. The model was tested against experimental measurements (Fig 3).

Though the model is not immediately relevant to estuaries there is no inherent difficulty in extending the numerical techniques to non uniform, unsteady conditions.

Kerns et al (1979) have developed a multi-layer, 1-D model of this type which allows non uniform cross-sections to be considered but it has apparently only been applied under steady flow conditions. The basic flow equation

$$\frac{dL}{dx} + \frac{dC}{dx} + \frac{dE}{dx} = \frac{dU}{dx}$$  (20)

where

- $d$ is water depth
- $h_B$ is elevation of the bed
- $C$ is Chezy coefficient
- $R_h$ is the hydraulic radius

is used to calculate the magnitude of the depth averaged current $U(x)$.

A logarithmic profile is assumed for the vertical structure of the flow. The suspended solids concentration is computed from the 1-dimensional form of the sediment concentration equation 2 using non uniform vertical grid to give greater accuracy near the bed. Kerns et al (1979) assumed the turbulent diffusivity, $D_T$, to equal the parabolic eddy viscosity (eq 8) appropriate for logarithmic flow. The boundary condition at the bed is taken to be the equilibrium concentration, equation 10, which would occur at the instantaneous flow conditions. The solution simulates the turbulent evolution of the equilibrium profile from a non equilibrium condition. The model was tested against infill rates measured in a gas pipeline trench in the Western Scheldt. Some results from the paper are reproduced in Fig 8.

Koutitas and O'Connor (1980) present a similar model, but use the alternative bed condition (12) instead of prescribing an equilibrium concentration at the bed. Their paper is mainly concerned with the mathematics of finite element versus finite difference solutions. They show that the qualitative structure of the results are not sensitive to the computational method used but the full finite element model proved to be more expensive than the preferred hybrid finite element/
Fig. 4. Comparison of observed plume structure in Apalachicola Bay. (From Fig. 6 in Wilson (1979), courtesy Academic Press Inc. (London) Ltd.)

Fig. 5. Comparison of observed and calculated concentration profiles. (From Fig. 3 of Valin and Finklenson (1973), courtesy International Association for Hydraulic Research)

Fig. 15. Development of suspension structures over one tidal cycle. (From Fig. 2 of Parker and Kirby (1976), courtesy BIFRA Fluid Engineering)

when the lateral spread of the plume is considered to be more important than the vertical profile. Wilson (1979) describes a model of this sort with an integral solution representing the balance between advection with the ambient current, lateral and longitudinal diffusion and settlement of particles of various sizes. The model is presented as a tool for studying sediment plumes produced from dredging losses. A typical comparison between results from the model and observations is reproduced in Fig. 4. Another integral solution for studying plumes from continuous discharges is presented by Christodoulou et al. (1974), and Chen et al. (1978) present a special model which computes bed changes in a reservoir due to settlement of suspended material carried in by the incoming water. The water flow in the latter case is obtained from the classical structure of a turbulent jet, and the suspended solids concentrations are computed using a finite difference scheme. Chen et al. (1976) also present some experimental results which could be of value in testing models of this type.

Fig. 5. Development of suspension structures over one tidal cycle. (From Fig. 2 of Parker and Kirby (1976), courtesy BIFRA Fluid Engineering)

near the bed to balance the seaward drift entrained by the surface water. Special techniques will again be required to study the transport of mud in these situations.

The simulation of mud transport requires a more comprehensive knowledge of the physical properties of the sediment than was required for studying sand transport. The properties of mud, such as settling velocity, erosion stress, deposition stress and erosion rate have been the subjects of many experimental studies over the years (Farthing (1965), Krone (1962), Owen (1971), and Thorn and Parson (1980)).

It appears that the settling velocity of cohesive material in suspension for concentrations less than 3000mg/l is proportional to the concentration with a proportionality factor of between 0.001 and 0.003m/s per mg/l. This is because there is a greater likelihood of collisions when there are more particles present. Above 5000mg/l the settling velocity does not increase linearly because of hindered settling.

If the local bed stress is too high mud particles or fines which drop onto the bed cannot settle. Krone (1962) postulated that the probability, P, that fines stick to the bed increases linearly from zero as the bed stress, $r_b$, falls below the critical deposition stress, $r_d$, thus

$$ P = 1 - r_b / r_d $$

$$ r_b < r_d $$

(30)

The net deposition rate can then be expressed as

$$ S_d = P \tau_0 \tau_c (1 - r_b / r_d) $$

(31)

with the value of $r_d$ determined from tests carried out on mud from the estuary under investigation.
It is well known (Ananthakrishna and Kroene (1976) and Thorn and Parsons (1980)) that erosion does not commence until the bed shear exceeds a certain critical value, \( \tau_b \), and that the erosion rate seems to be proportional to the excess shear. However, this critical stress varies with the density of the exposed mud surface, which in turn varies with time. As mud is eroded a more consolidated (and hence denser) mud is exposed which requires a corresponding higher bed stress to put it into suspension. Farinhalde (1965) proposed representing erosion as

\[
\varepsilon = M(\tau_b - \tau_b) \quad \tau_b > \tau_b
\]

in which the erosion rate \( M \) has to be prescribed from experiments in addition to the critical erosion stress.

The fundamental equation governing the transport of mud in suspension is identical to equation 2 with the source/sink term given by equations 31 or 32. The special nature of these boundary conditions at the bed precludes the formulation of analytic solutions for all the models described below are numerical. The models reported in the literature are often designed to meet the particular requirements of certain problems. In the following models are separated into those which involve just one horizontal dimension in which lateral effects are neglected, and those which take into account lateral as well as longitudinal variations. Either sort of model can include a description of vertical effects.

### 4.1 One-dimensional mud models

One-dimensional models are designed for estuaries or tidal channels where lateral variations are not important. Unfortunately those situations are likely to be more affected by salinity effects insofar that secondary currents can be created in the vertical plane. Even if the mud appears to be well mixed through the water column the presence of non-logarithmic velocity profiles necessitates the use of 2 or more layers in the vertical to simulate the mud transport.

The earliest model of this type presented by Odé and Owen (1972) used 2 layers to simulate the mud transport processes in the Thames Estuary. The lower layer was given a fixed thickness and the flow and concentration calculated in each layer using a finite difference method. The exchange of material between the lower and the bed was based on relations of the type given in equations 31 and 32 in which the critical stresses were estimated from laboratory experiments. Exchange of material between the upper and lower layer was computed from the net flux of mud due to settling, upwelling and vertical mixing by turbulence. The model was verified by comparing predicted and observed concentrations. The results showed very good agreement between areas of silting predicted by the model and the existing, muddy reaches of the river (Fig. 16). The application of the model to study the effects of a tidal barrage clearly demonstrated its value for the engineer.

This model was subsequently improved to allow the interface between the layers to adjust dynamically to the flow and salinity variations. It has been applied to study tidal flows, saline intrusion and sediment transport in the Rotterdam Waterways. A series of sensitivity tests was carried out to demonstrate the effect of altering mixing and entrainment coefficients on the pattern of saline intrusion. Comparison with two independent sets of field observations showed that the model simulated all the gross features of the pattern of saline intrusion in the waterways for average and low flow conditions with the same set of model parameters. The model also simulated correctly all the important periodic variations in the concentrations of suspended mud in both layers during a period of low flow conditions.

\[
\frac{w_c + D_z B}{2} = 0 \quad \text{at} \quad z = d
\]

There are two possible conditions at the bed which admit analytical solutions. Firstly one could assume (Mei 1969) that the concentration \( c(t) \) at the bed responds instantly to the changing flow conditions.

That is

\[
c(t) = c_{eq} \quad \text{when the flow is saturated with sediment}
\]

\[
c_{eq} = \frac{c(t)}{t}
\]

\[ \rho_f = \text{a profile factor} \]

This is a much more realistic condition than that implicit in a potential flow model which assumes that the full load response instantly. However, the condition still implies an infinite rate of exchange of material at the bed at \( t = 0 \) (or at \( x = 0 \) in the non-uniform version).

Lee (1980) assumes that the rate at which material is entrained into the flow is the quantity which responds most readily to changes in flow. In this case the boundary condition at the bed would be

\[
\frac{D_z}{D_z} \mu_{eq} = 0 \quad \text{at} \quad z = B
\]

or, from equation 4, the net vertical flux at the bed is prescribed as

\[
\mu_{eq} = \mu_{eq} = w_d \rho_f \rho_f - \tau_f
\]

This provides an alternative boundary condition to condition 8. The asymptotic form of solutions to equation 5 for initial concentration \( c_{eq} \) are

\[
\begin{align*}
\phi(t) &= \frac{2}{\rho_f} \rho_f^2 \rho_f^{-2} + \frac{1}{2} \rho_f \left( c_{eq} - \rho_f \right) \left( \rho_f \rho_f + \rho_f \right) + \phi(0) \\
\phi(t) &= \frac{2}{\rho_f} \rho_f^2 \rho_f^{-2} + \frac{1}{2} \rho_f \left( c_{eq} - \rho_f \right) \left( \rho_f \rho_f + \rho_f \right) + \phi(0)
\end{align*}
\]

where

\[
R = \frac{w_{eq} \rho_f}{\tau_f}
\]

\[ \tau = \left( \frac{w_{eq} \rho_f}{\tau_f} \right)^{1/2} \]

\[ \sigma = \left( \frac{w_{eq} \rho_f}{\tau_f} \right)^{1/2} \]

\[ z = \rho_f \]

for the bed concentration boundary condition (Mei) and the bed entrainment boundary condition (Lee) respectively. These solutions are valid when \( \tau < 1 \), i.e for times while water flows over distances equal to 10 to 100 water depths.

There is another class of special analytic solutions which are related to the plume models used for thermal or sewage pollution studies. These models are usually depth integrated and are designed for situations
This equation represents the equilibrium profile obtained as a balance between settling and vertical diffusion due to the turbulence. It is important to appreciate that equilibrium defined in this way does not mean saturation. Indeed the sediment load can be in equilibrium if the bed is not mobile, even when the flow is undersaturated with sediment. Integration with respect to z and the application of a boundary condition of zero flux of sediment at the free surface (and implicitly also at the bed) yields the governing equation

$$\gamma w + D_\alpha \frac{\partial \gamma}{\partial z} = 0$$

(4)

This can be integrated further if the vertical structure of the diffusivity is prescribed. These profiles have proved valuable in the understanding of sediment transport but they are not relevant to the unsteady, unsaturated flow conditions which are the main concern here, so these special solutions are not considered further. Graf (1971) is a good source of additional information on these solutions.

A class of solutions which have more relevance to estuaries have been presented by Kalinske (1940), Dobbins (1943), Mei (1969) and Lean (1980), for unsteady and uniform or steady and non-uniform conditions governed respectively by the equations

$$\frac{\partial \delta}{\partial t} + \frac{1}{\gamma} \frac{\partial \delta}{\partial x} + \frac{\partial (\delta \gamma)}{\partial x} = 0$$

(5)

$$\gamma w = -2 \gamma \frac{\partial \delta}{\partial t}$$

(6)

These equations are mathematically the same when $$\gamma$$ is constant. The first represents a concentration changing with time following a change in the magnitude of a uniform flow, while the second represents the concentration changing as a function of position as might occur for example when clear water flows from an area with an ineradicable bed into an area where erosion can commence. Solutions of these equations provide information about the time or distance of travel required for the sediment concentration to adapt to changes in the flow conditions.

The assumption of constant eddy diffusivity permits analytical solution of these equations. The diffusivity normally used is

$$D_\alpha = \frac{1}{k} \frac{\partial \gamma}{\partial \delta}$$

(7)

which is the depth averaged value of the parabolic eddy viscosity

$$\nu_e = k \frac{\partial \gamma}{\partial \delta} (1 - \delta)$$

(8)

consistent with the logarithmic velocity profile. $$k$$ is the Von Karman constant. Apsman and Rutter (1970) present experimental evidence that supports this assumption. The exact solutions, which involve the use of Laplace Transform, or similar, may be expressed in the form of infinite series. Mei (1969) recognised that an approximate solution, valid for small times or distances of travel, could be obtained from the expansion of the Laplace Transform for large values of the transform parameter. Under most conditions this expansion is valid for distances of the order of twenty water depths or the equivalent in a time dependent condition. Lean (1980) proposed an alternative bed boundary condition and the present author has reworked Mei's solution for this case. This solution is used in the sediment transport model proposed in the next section.

The solution of the unsteady equation requires an initial condition at $$t = 0$$ and boundary condition at $$z = 0$$ (the bed) and $$z = d$$ (the free surface). The surface boundary condition is clearly zero vertical flux of sediment viz

$$w = 0$$

(9)

4.2 2-dimensional mud transport models

Boonville (1976) has applied a very similar 2-layer model with a constant lower layer to the Gironde.

Experience in the use of these 2-layer models at the Hydraulics Research Station has identified the need for and led to the development of numerical techniques for describing the vertical structure of the flow with many layers. A model of this type has been applied to study the situation in the extensions to the Port of Brisbane. The essentials of the model are described by Rodger (1980). The model uses relations of the form in equations 31 and 32 for exchanges of mud between the layers and the lower layer, and includes a layered description of the bed to keep account of the age of deposits. Exchange of material between the layers is simulated as a balance between settling, upwelling and turbulent mixing as for the earlier 2-layer model but in addition the rate of turbulent mixing is adjusted dynamically to allow for stratifications in the flow as described by Odd and Rodier (1979).

Watanabe et al (1978) propose a different method of representing the vertical profile using a finite element approach. At present this model is limited to uniform flow but the paper contains some flume data which could serve as a useful check on model response to changes in flow.

A model to study a special fluid mud problem in the River Avon was developed at the Hydraulics Research Station (HRS Ex 703 (1975)). Mud concentrations in this river exhibit the same type of behaviour described by Fisher and Kirby (1970) for the Severn Estuary into which the river in fact flows. The model was designed to simulate three distinct phases of suspension — a static bed layer with a uniform concentration of 150 000 ppm, a fluid mud layer with concentrations varying between 75 000 and 10 000 ppm and a suspended mud layer with a variable concentration distributed uniformly through the layer. The flow and transport were calculated dynamically using a finite difference technique. The exchange relationships between the layers were similar to those used by Odd and Owen in their 2-layer model. The erosion rate was chosen during calibration to produce the correct thickness of static layer observed in the river. The model was used to study the influence of a proposed barrage on the mud regime of the River Avon.

The extension of the numerical techniques used in the 1-dimensional models to study mud transport in 2-horizontal directions were not yet reached the stage for presentation in the literature. One reason is undoubtedly because of the high cost of 2-dimensional layered models...
on the computer but there is also the extra difficulty in defining the properties of the bed over an area. To date the only known models for two-dimensional areas are in fact solutions of the model formulation equation for the mud concentration, \( C(x, y) \):

\[
\frac{\partial C}{\partial t} + \frac{\partial (v_C D_C u_C)}{\partial x} + \frac{\partial (v_C D_C v_C)}{\partial y} = \frac{\partial}{\partial x} \left( \frac{D_C}{\theta_C} \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{D_C}{\theta_C} \frac{\partial C}{\partial y} \right) + S
\]

(33)

where \( S \) is defined by relations of the form given in equations 31 and 32. The main difference between this equation and equation 31 is that no profile parameters are considered necessary when very fine sediments are involved. Models based on equation 33 are therefore mainly valid in situations where the suspended material is well mixed in the vertical (concentrations < 5000 mg/l) and when lateral effects are more important.

The only published model of this type (Arahata and Krone (1976)) makes use of a finite element technique. The rate of exchange between the flow and the bed is simulated by relations like those given in equations 31 and 32. The model was tested by comparing predicted siltation against siltation measured in a flume experiment (Fig. 17). The experimental setup consisted of a permeable barrier to create non-uniformities in a steady flow. Siltation occurred in the shelter of the barrier. The potential of the model was demonstrated by considering various alternatives for a hypothetical harbour problem.

3 SUSPENDED SAND TRANSPORT MODELS

The potential load model is not suitable for studying sand transport in situations where there is not a continuous supply of erodible material on the bed. The reason is that that sort of model cannot take into account where the water carrying the sediment has been nor how much sediment is actually being carried by the flow. To do these requires a different sort of model based on conservation principles which simulates the sediment transport in terms of a suspended solids concentration. The erosion or deposition of material on the bed can then be assumed in the model depending on whether the actual load is less or greater than the saturated load which would obtain under steady, uniform flow conditions at the same values as the instantaneous flow. This assumption effectively ignores the differences in turbulence which occur in accelerating or decelerating flow. Under these circumstances the suspended solids concentration, \( c \), (kg/m³) satisfies (eg Graf (1971))

\[
\frac{\partial c}{\partial t} + \frac{\partial c}{\partial x} u_C + \frac{\partial c}{\partial y} v_C + \left( w_c - \frac{w_c R_c}{g} \right) = \frac{\partial}{\partial x} \left( \frac{c R_c}{\theta_C} \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{c R_c}{\theta_C} \frac{\partial c}{\partial y} \right) + S
\]

(2)

where

- \( u, v, w \) are the velocity components
- \( x, y, z \) are space co-ordinates, with \( z \) vertically upwards
- \( D_x, D_y, D_z \) are the (turbulent) diffusion coefficients
- \( w_c \) is the settling velocity
- \( S \) is the sink or source term representing erosion or deposition of material on the bed
- \( t \) is time.

Most solutions to be found in the literature are for special cases of this equation. The earlier solutions by Schmidt (1925) and Lane et al (1941), and later Hunt (1965) are essentially geared to providing insight into the vertical structure of the suspended solids profile. These assume one-dimensional, uniform, steady flow conditions for which equation 2 reduces to

\[
\frac{\partial c}{\partial z} + \frac{\partial}{\partial z} \left( \frac{c R_c}{\theta_C} \frac{\partial c}{\partial z} \right) = 0
\]

(3)
Ouss proved most successful because the bed contained an adequate supply of erodible sand and inertial lag effects were small. Using a model of the river it was possible to reproduce the evolution of bed level profiles over a period of two years (Fig 3) and predict the changes which would occur following the construction of a tidal barrage and/or extracting fresh water. A second application to a 2-dimensional area in the south-east corner of the Wash was less satisfactory. The complicated nature of outfall channels prevented the collection of enough data to calibrate the model properly. Nevertheless it was still possible to use the model to predict the changes in sediment transport patterns which could occur following the construction of reservoir schemes.

Fig 3. Bed levels and flow in the Great Ouse

Recently Katoh et al (1979) have presented a 2-dimensional potential load model which can take into account several distinct sand sizes. According to the English synopsis of their paper they have been able to forecast long-term bed changes for a site in the Ariake Sea. The predicted bed changes were apparently fed back into a complementary flow model without causing stability problems.

The next application by the Hydraulics Research Station was made to provide information about the lateral distribution of siltation in the Brisbane River to complement the longitudinal distribution of silting predicted by the HRS multi-layer model. The work was commissioned by the Port of Brisbane Authority. An outline of the study was presented by Odd and Baxter (1980) and the detailed analysis of the 2-dimensional model outlined in Ref 14. The 2-dimensional model was run under steady peak flood and ebb flows on the principle that deposition under these conditions would identify areas where mud could accumulate. The model was verified against cores collected from areas identified by the model to be muddy or not muddy (Fig 18). The siltation rate was calculated from a reference to known dredging rates under existing conditions and the model used to predict areas and rates of siltation in the new docks.

Fig 18. Bed-stress distribution in the Fisherman Islands Swing Basin

The most recent application was to study silting aspects of proposed engineering works in the Conway Estuary (Ref 15). The 2-dimensional models were in this case run over spring tidal cycles and again deposition was governed by relation 31. There was no information available on the erosion properties of the Conway mud and consequently the effect of erosion was taken into account by choosing a critical stress for the accumulation of deposits. This critical stress was determined by comparing the maximum bed stress contours (predicted by the model) against the existing muddy areas. The same critical stress was subsequently used to identify the possible advance or retreat of the muddy areas following the construction of works. There were no siltation rate or dredging records available from the estuary to calibrate the model. Accordingly siltation rates in the developed state were presented in the form shown in Fig 19 as ratios relative to the existing (but unknown) siltation rate at the same point. This information was used by environmentalists to assess the impact of the works on the ecology as well as by the engineers to assist in the design of works.
5 STATE OF SEDIMENT TRANSPORT MODELLING

From this review of published information it is clear that the expertise already exists to formulate and develop the appropriate numerical techniques. Furthermore computers are becoming more powerful and machines exist which can cope with the escalating number of calculations involved as the models become more sophisticated. The main factor limiting the advance of numerical sediment transport models is probably the uncertainties that exist about the physical relations which have to be fed into the models. Table 3 shows a list of the main relations that influence sediment transport in estuaries. The quality of the relation is graded from very good to poor merely to give a relative guide, but hopefully it will also inspire some researchers to improve the situation.

6 CONCLUSIONS

Many numerical models for studying sediment transport have been presented in recent years but very few are suitable for application to estuaries. If the author’s experience is typical, it is likely that many engineering studies have been made that have not been published because of pressures of other work or because the models were not fully calibrated or verified. From a scientific point of view an unverified model is of very little value. However this is not necessarily true in an engineering sense provided the modeller or engineer recognises the shortcomings of the model and takes care in interpreting the model results. Results should not be taken on their face value but used to supplement the experience of the engineer, and to pinpoint the areas that the engineer should concentrate his attention on. Used in this way even an unverified model is a valuable tool. For this reason there is a need for modellers and engineers to publish more readily their sediment transport case studies. Other workers in the field will appreciate the reason for limitations in such studies and the ensuing discussion can only serve to improve sediment models generally in the long-term.
In the following report various methods are described which have appeared in the engineering journals and proceedings of conferences over the last few years. The emphasis is on the formulation of models and applicability rather than on the details of particular numerical schemes used to obtain the solutions. We have tried to treat the methods in a systematic way starting with those appropriate for bed load transport and ending with models for studying fine all and mud transport in suspension. There is inevitably some overlap between the modelling techniques when fine sand is concerned but it is hoped that the limits of applicability of any particular model will be clear. The author has attempted to fit all the relevant papers and reports known to him into this review, and apologises for any omissions which may have occurred through oversight.

2 POTENTIAL LOAD MODELS

The simplest type of sediment transport model is essentially a single equation representing conservation of bed material.

\[ \frac{\partial m}{\partial t} + \frac{\partial (m \bar{u})}{\partial x} = 0 \]  

(1)

where \( m \) (kg/m\(^2\)) is the quantity of material on the bed and \( \bar{u} \) (kg/sec/m width) is the sand transport. The basic assumption for this type of model is that the flow is saturated with sediment, which means that the flow is carrying the maximum sand transport that can be maintained for the given hydraulic and sedimentary conditions. Under saturated conditions the transport can be calculated from one of the many sediment transport laws to be found in the literature. An appraisal of available methods is given by White et al (1973). The flow parameters (water depth, mean velocity and shear velocity) required for the transport calculation could be obtained from measurements in a physical model but it is usually quicker and cheaper to generate this data on a routine grid from a separate numerical model of water movements.

The sediment carrying capacity of flow increases significantly for high water velocities -- typically in proportion to the fourth power. This means that the flow will tend to pick up material from the bed when it accelerates and to deposit excess material when it decelerates. If the flow is always saturated with sediment the differences in transporting capacity must define the quantity of material picked up or deposited on the bed. This is the basis for the potential model load. The computer is only used because it is many times faster than calculating by hand. In this way the sand transport calculation can be made for hundreds of points and repeated at intervals to define the variation of sand transport over the estuary and through the tidal cycle.

The potential load model is naturally most suited to situations where the bed material is narrowly graded and where there is an adequate supply of erodible material on the bed to maintain the saturated load. These conditions are more often met in rivers and it is in such situations that potential load models have been found most successful. See for example Concus and Perdrew (1973), de Vries (1976), Thomas and Prasuhn (1977) and Bettess and White (1979). An example of the results from the last of these is given in Fig 1.

Lepetit and Hague (1978) have extended the modelling approach used in river studies, to simulate 2-dimensional local scour round a jetty in a steady flow. The model is quasi-steady and uses a perturbation technique to feed the changes in depth back into the flow. Transport is calculated from a saturated bed load sediment law and bed changes calculated from the 2-dimensional form of equation 1, for conservation.
1 INTRODUCTION

An estuary is a partly enclosed body of tidal water where river water is mixed with and diluted by sea water. In a general sense the estuarine environment is defined by salinity boundaries rather than by geographical ones, but although the salinity has influence on the clay sediment fractions it is the currents generated by the tidal volume flowing in and out of the estuary which dominate the movement and distribution of sediments. The sediments themselves may have originated from natural erosion inland or from seawards. They consist of materials ranging from the finest clay particles to coarse sand and gravels. A convenient classification of sediments uses a geometric scale of sizes

<table>
<thead>
<tr>
<th>Grain Type</th>
<th>Medium</th>
<th>phi units</th>
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</thead>
<tbody>
<tr>
<td>Very coarse sand</td>
<td>1.0 - 2.0</td>
<td>-1</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>0.5 - 1.0</td>
<td>0</td>
</tr>
<tr>
<td>Medium sand</td>
<td>0.25 - 0.5</td>
<td>1</td>
</tr>
<tr>
<td>Fine sand</td>
<td>0.125 - 0.25</td>
<td>2</td>
</tr>
<tr>
<td>Very fine sand</td>
<td>0.064 - 0.125</td>
<td>3</td>
</tr>
<tr>
<td>Coarse silt</td>
<td>0.032 - 0.064</td>
<td>4</td>
</tr>
<tr>
<td>Medium silt</td>
<td>0.016 - 0.032</td>
<td>5</td>
</tr>
<tr>
<td>Fine silt</td>
<td>0.008 - 0.016</td>
<td>6</td>
</tr>
<tr>
<td>Very fine silt</td>
<td>0.004 - 0.008</td>
<td>7</td>
</tr>
<tr>
<td>Coarse clay</td>
<td>0.002 - 0.004</td>
<td>8</td>
</tr>
<tr>
<td>Medium clay</td>
<td>0.001 - 0.002</td>
<td>9</td>
</tr>
</tbody>
</table>

**TABLE 1 SEDIMENT GRADINGS**

A significant feature of estuaries is the wide range of sediment sizes found in them. These sediments are sifted and sorted by the tidal currents.

In the main channels bed stresses are usually too high to allow the finer materials to accumulate although they may settle temporarily at slack water. Only coarse sand and gravel can exist as permanent deposits in these high energy regions. Along the shallow margins of the estuary, and further upstream, the tidal currents are too weak to move the sand and either no sand is transported there or it is covered by silt or clay to produce characteristic mudflats. These mudflats are colonized by various forms of marine life and become the feeding grounds of birds. If conditions are suitable the level of the mudflats rises and eventually a salt marsh develops.

The study of sediment transport generally is a very difficult problem. The particular study of sediment transport in estuaries is especially complicated because

1. The water movements are continually changing with the rise and fall of the tide;
2. The wide range of sediments present in suspension and on the bed;
3. The absence of certain sediments in some parts of the estuary leading to unsaturated sediment loads in the water.

Although it is not possible to predict precisely how any single type of sediment will behave in the estuary, the recent advances in numerical modelling do enable some information to be obtained to give guidance to engineers and environmentalists for assessing the impact of engineering works.
SUMMARY

This paper is a review of numerical models for studying sediment transport with special emphasis on the applicability of methods to estuarine conditions. The main features of estuaries are the wide range of sediments present, the absence of erodible material in some places, a combination of unsteady and nonuniform flow and lateral as well as longitudinal variations in the flow and suspended solids concentrations. Saline stratification can also be present. Not surprisingly no single model has so far been presented to simulate all the estuarine sediment processes but many models are described which can be used to study certain aspects. The models are separated into potential load models primarily geared to studying bed load transport, suspended sand models and suspended mud models.

The review covers the most significant papers published in recent years, supplemented by some case studies of projects carried out in the Hydraulics Research Station, Wallingford.

9 LIST OF SYMBOLS

$\phi(x,y,z,t)$: concentration of suspended solids (kg m$^{-3}$)

$\phi(x,y,z,t)$: depth-integrated concentration (kg m$^{-2}$)

$c_0$: initial concentration (kg m$^{-3}$)

$c_s(x,y,t)$: saturation ratio, $c/c_0$

$c_s$: concentration under saturated conditions

$c_s$: depth-integrated, saturated concentration

$C$: Chezy friction factor

$d$: water depth (m)

$D$: sediment grain size (mm)

$D_a$: lateral (turbulent) diffusion coefficient (m$^2$ s$^{-1}$)

$L_T$: longitudinal (shear) dispersion coefficient (m$^2$ s$^{-1}$)

$L_x$, $L_y$, $L_z$: diffusion coefficients in 3-D axes (m$^2$ s$^{-1}$)

$E$: sand transport coefficient (kg m$^{-3}$ s$^{-1}$)

$F_z$: net vertical flux of sand (kg m$^{-2}$ s$^{-1}$)

$g$: acceleration of gravity (m s$^{-2}$)

$m$: quantity of mobile bed material (kg m$^{-2}$)

$M$: erosion rate (kg m$^{-3}$ s$^{-1}$)

$n$: natural co-ordinate normal to flow

$P$: probability of deposition
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- $q$: speed of current, $(U^2 + V^2)^{1/2}$ (m$^3$/s$^2$)
- $Q$: discharge per unit width (m$^3$/s$^2$)
- $R$: settling velocity Reynolds Number, $w_d/2D_g$
- $R_h$: hydraulic radius
- $s$: natural co-ordinate in direction of flow
- $S$: source/sink term at the bed (kg m$^{-2}$ s$^{-1}$)
- $S_d$: deposition rate (kg m$^{-2}$ s$^{-1}$)
- $S_e$: erosion rate (kg m$^{-2}$ s$^{-1}$)
- $S_a$: vertical flux of sand (kg m$^{-2}$ s$^{-1}$)
- $t$: time
- $T_d$: sand transport (kg m$^{-2}$ s$^{-1}$)
- $(u, v, w)$: components of velocity vector (ms$^{-1}$)
- $(G, T)$: depth integrated velocity components (ms$^{-1}$)
- $u_{crit}$: threshold velocity for sand transport (ms$^{-1}$)
- $v_0$: uniform velocity (ms$^{-1}$)
- $w_d(D)$: settling velocity (ms$^{-1}$)
- $(x, y, z)$: cartesian co-ordinates
- $z_b$: elevation of bed (m)
- $a$: profile parameter, $f_{ac}/d_x$
- $b(r)$: bed exchange factor
- $b_k$: profile parameter, $c(x, y, o, 0)/c(x, y, 0)$
- $\kappa$: Von Karman constant
- $\nu$: eddy viscosity (m$^2$/s$^2$)
- $\rho_b$: consolidation ratio of deposited mud (kg m$^{-2}$)
- $\omega$: dimensionless variable, $(w_d^2/4D_g)^{1/2}$
- $\tau$: dimensionless variable, $(\omega^2/4D_g)^{1/2}$
- $\tau_b$: bed stress (Nm$^{-2}$)
- $\tau_d$: critical stress for deposition (Nm$^{-2}$)
- $\tau_e$: critical stress for erosion (Nm$^{-2}$)
- $\delta$: dimensionless vertical co-ordinate, $z/d$
SEDIMENT TRANSPORT MODELS FOR ESTUARIES
G V Miles

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Hydraulics Research Station
Wallingford
Oxon OX10 8BA
Telephone 0491 35381